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CENTRE D'ORSAY

LABORATOIRE DE PHYSIQUE

DE L'ECOLE NORMALE SUPERIEURE

THESE

présentée pour obtenir

le grade de DOCTEUR ES-SCIENCES

par

M. Costas KOUNNAS

Sujet : "Prédictions de la chromodynamique quantique au deuxième ordre"

Soutenue le 18 décembre 1981 devant la Commission d'examen

MM. Ph. MEYER, Président
C. BOUCHIAT, Rapporteur
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Mme C.A. PIKETTY
MM. J. PRENTKI
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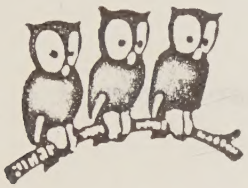

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- I -

INTRODUCTION

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Un des progrès les plus remarquables dans la physique des particules élémentaires pendant ces dernières années a été indiscutablement la découverte que les nucléons, loin d'être des particules "élémentaires", possèdent une structure interne. Cet exploit a été réalisé surtout grâce aux expériences qui font intervenir des leptons, électrons, muons ou neutrinos. Il s'agit de la diffusion à haute énergie et à grand moment de transfert des leptons sur les nucléons appelée "diffusion profondément inélastique" des processus d'annihilation électron-positron, ou bien de la création d'une paire lepton-antilepton. Du point de vue théorique, il existe deux façons d'approcher ce problème. Dans la première, on suppose que le nucléon est composé à partir des constituants élémentaires qu'on appelle "partons". Les partons interagissent avec le courant (électromagnétique ou faible) comme des charges ponctuelles et on néglige toute interaction entre eux [1]. Ce modèle naïf a connu un grand succès parce que (i) il prédit l'invariance d'échelle [2] qui est observée avec une bonne approximation et (ii) il présente une image simple et permet de donner une interprétation physique claire aux différentes quantités qu'on calcule. Cependant il a plusieurs défauts, le principal étant que les interactions fortes entre les partons, qui sont responsables, entre autres, du confinement de ceux-ci, ont été négligées sans justification. En plus, sur le plan expérimental, le modèle des partons ne permet pas d'expliquer les faibles violations de l'invariance d'échelle, qui sont pourtant observées. Pour toutes ces raisons, une seconde approche a été développée, qui est basée sur la théorie quantique des champs.

Le point essentiel est une propriété remarquable des théories de jauge non abéliennes d'être "asymptotiquement libres" [3]. Ceci signifie que leur intensité varie avec la distance: négligeable à très courtes distances (d'où le terme

"liberté asymptotique", elle devient très forte, peut-être infinie, lorsque la distance de séparation augmente. Ainsi les constituants des nucléons, les partons, se comportent comme des particules libres lorsqu'on les "regarde" avec des sondes de très haute énergie, donc à des distances très courtes, tout en restant très liés, peut-être par des forces infinies, à l'intérieur des nucléons. On suppose donc que les interactions fortes sont décrites par une théorie de jauge du groupe SU(3) de la couleur (chromodynamique quantique) [4]. La dynamique des réactions est séparée en deux régimes [5,6,7,8] : (i) celui des petites distances, qui, grâce à la propriété de la liberté asymptotique, est gouverné par de faibles constantes de couplage et peut être calculé en utilisant la théorie des perturbations et le groupe de renormalisation [9] et (ii) celui des grandes distances, qui est responsable de la formation des hadrons à partir des partons et qui ne peut qu'être paramétrisé à l'aide des fonctions qu'on détermine expérimentalement. La stratégie typique d'une telle analyse consiste à déterminer la partie non calculable des grandes distances à l'aide des résultats expérimentaux à une énergie et d'utiliser par la suite cette information afin de prédire les résultats d'autres expériences. Cette comparaison peut, en principe, être poussée à un ordre arbitraire de la théorie des perturbations, mais, en fait, dans la plupart des cas, les théoriciens se limitaient au calcul du premier ordre non trivial [10]. Néanmoins, cette théorie, avec un minimum de paramètres, rendait bien compte de tous les résultats expérimentaux, mais en revanche, l'image intuitive simple qu'on avait avant était sacrifiée. La sophistication des techniques utilisées, telles que le groupe de renormalisation [9], le développement de Wilson d'un produit d'opérateurs [5], ainsi qu'un formalisme plus récent dit "des vertex coupés" [7,8], contrastait avec la simplicité des concepts du modèle des partons.

Dans cette thèse, j'ai entrepris une approche globale de ces problèmes. En premier lieu, en collaboration avec mes collègues, j'ai réussi à construire un modèle des partons généralisé [11,12] qui tient compte, d'une façon systématique,

des interactions fortes décrites par la chromodynamique quantique. En particulier, nous avons démontré à tous les ordres de la théorie des perturbations que les fonctions qui paramétrisent notre ignorance sur la dynamique des grandes distances, ont une interprétation physique très simple : elles décrivent la distribution des partons (quarks, antiquarks ou gluons) à l'intérieur de chaque hadron et la probabilité de fragmentation de chaque parton en hadrons. En outre, ces fonctions sont universelles, elles décrivent des propriétés intrinsèques des hadrons. Elles représentent la structure du hadron considéré, et, par conséquent, elles ne dépendent pas de la nature de la sonde qu'on utilise mais seulement de son pouvoir de résolution. Leur valeur absolue n'est pas calculable car il s'agit d'un phénomène de grandes distances. En revanche, leur variation en fonction de l'énergie incidente du courant qui représente la sonde l'est. Par contre, la dynamique à courtes distances est donnée uniquement en termes d'une charge effective de chaque parton qui dépend du courant incident, mais non du hadron à l'intérieur duquel se trouve le parton. Ces charges sont calculables en théorie des perturbations. Nous sommes ainsi amenés à exprimer toutes les grandeurs mesurables en termes de quantités qui ont une interprétation probabiliste compatible avec la notion du parton et équivalente à la méthode standard de la théorie des champs.

Ensuite, nous nous sommes attaqués au problème de la contribution du deuxième ordre. En effet, le succès, quantitatif et qualitatif, de la chromodynamique quantique au premier ordre de la théorie des perturbations [10], tout en étant impressionnant, laissait plusieurs questions sans réponse.

En particulier, la constante de couplage n'était pas déterminée sans ambigüité et les termes d'ordre supérieur ne pouvaient pas être considérés comme négligeables. La précision des résultats expérimentaux était déjà telle que cette question prenait un caractère d'urgence. D'autre part, il a été

remarqué que les quantités calculées dans la région de moment de transfert du genre espace et celle du genre temps étaient liées par des relations qu'il serait intéressant d'approfondir. Malheureusement, au premier ordre, deux relations indépendantes de ce genre étaient simultanément satisfaites, l'une due à Gribov et Lipatov [13] et l'autre qui était, essentiellement, une relation de prolongement analytique [14]. En se limitant aux calculs connus, il n'était pas possible de décider laquelle, s'il y en avait une, avait une signification profonde et laquelle était due à un accident du premier ordre.

Nous avons donc commencé un calcul systématique de tous les effets du second ordre aussi bien pour les processus dans la région du genre espace que celle du genre temps. Il s'agit d'un travail d'une assez grande complexité technique, ainsi nous avons essayé d'abord nos méthodes de calcul sur un modèle simple, qui présente quelques similitudes avec la chromodynamique quantique sans en avoir toute la complexité. C'est une théorie d'un champ scalaire neutre, en interaction donnée par un terme ϕ^3, dans un espace-temps de six dimensions.[12,15] Encouragés par nos résultats, nous avons procédé aux calculs de la chromodynamique quantique [17,18,19,20] proprement dite et dont les résultats sont aussi inclus dans cette thèse. Nous avons proposé une méthode simple d'analyse et de comparaison entre les prédictions théoriques et les résultats expérimentaux et avons calculé tous les termes nécessaires. Finalement, nous avons démontré que la relation de Gribov-Lipatov ne survit pas au deuxième ordre, en revanche celle de la continuation analytique reste valable et permet de simplifier considérablement les calculs.

Nos résultats s'ajoutent à d'autres groupes de chercheurs [21] et font un ensemble complet de prédictions théoriques face à des mesures expérimentales dont la précision ne cesse de croître [22]. De cette confrontation, la chromodynamique quantique se confirme comme la théorie fondamentale des interactions fortes.

Le plan de cette thèse est le suivant:

Dans le chapitre II, nous définissons les processus étudiés et nous précisons notre notation. Nous donnons également une interprétation probabiliste des vertex coupés.

Dans le chapitre III, nous présentons le modèle des partons généralisé et nous donnons les équations vérifiées par les fonctions de structure et de fragmentation. La démonstration de ces équations sera présentée dans le chapitre IV.

Dans le chapitre V, nous démontrons l'équivalence du formalisme des vertex coupés et du développement sur le cône de lumière dans le cas de la diffusion inélastique, et nous vérifions cette équivalence au niveau des diagrammes à deux boucles. Nous donnons aussi un algorithme de calcul des ordres supérieurs et nous l'appliquons dans le cas de la théorie scalaire $[\phi^3]_6$.

Les calculs de la chromodynamique quantique sont examinés dans l'approximation des diagrammes à deux boucles dans le chapitre VI pour le cas non-singulet, et dans le chapitre VII pour le cas singulet.

Dans le chapitre VIII, nous développons une méthode numérique pour résoudre les équations intégrodifférentielles vérifiées par les fonctions de structure et de fragmentation. Nous donnons toutes les quantités nécessaires à l'analyse des prédictions théoriques, dans l'approximation des diagrammes à deux boucles. Nous étudions aussi les relations entre les quantités du genre espace et du genre temps, comme la continuation analytique et la violation de la relation de reciprocité de Gribov et Lipatov.

Dans le dernier chapitre (IX), nous présentons une méthode d'analyse simple, valable dans le cas non-singulet, qui évite les complications pratiques dûes au produit de convolution.

Dans l'Appendice A, nous présentons un exemple de calcul d'un dia-

gramme à deux boucles qui contribue au noyau de variation de probabilité ($P(x,\alpha)$ et $\tilde{P}(x,\alpha)$).

Dans l'Appendice B, nous donnons la liste des transformations de Mellin et des produits de convolution pour certaines fonctions clés, qui peuvent être utiles pour les calculs et l'analyse des ordres supérieurs.

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- II -

G E N E R A L I T E S

1. Fonctions de structure et fonctions de fragmentation.

Nous étudierons deux types de processus: i) la diffusion profondément inélastique et ii) l'annihilation semi-inclusive.

i) Diffusion profondément inélastique.

Nous considérons la diffusion d'un lepton sur une cible hadronique l (lepton incident) + h (hadron) $\longrightarrow$ l' (lepton sortant) + X, (fig. 1) où "X" représente l'état final hadronique.

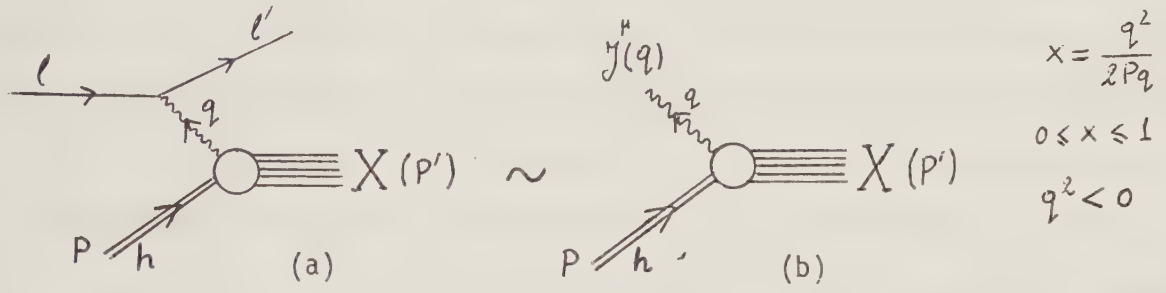


Fig. 1 : Diffusion inélastique. (a) l + h $\rightarrow$ l' + x (b) $\mathcal{J}$ + h $\rightarrow$ X

Dans l'approximation d'ordre le plus bas dans les couplages électromagnétique ou faible, il suffit d'examiner la diffusion d'un courant (électromagnétique ou faible) sur un hadron

$$J(q) + h(P) \longrightarrow X(P') \qquad (1)$$

Les variables cinématiques sont définies dans la figure (1b).

Il est alors utile de définir une fonction $W^{\mu\nu}$, qu'on appelle fonction de structure, de la façon suivante:

$$W^{\mu\nu}(x, q^2, P^2) \equiv \sum_X \int d^4y\, e^{iqy} \langle P | J^{+\mu}(y) | X \rangle \langle X | J^{\nu}(0) | P \rangle \qquad (2a)$$

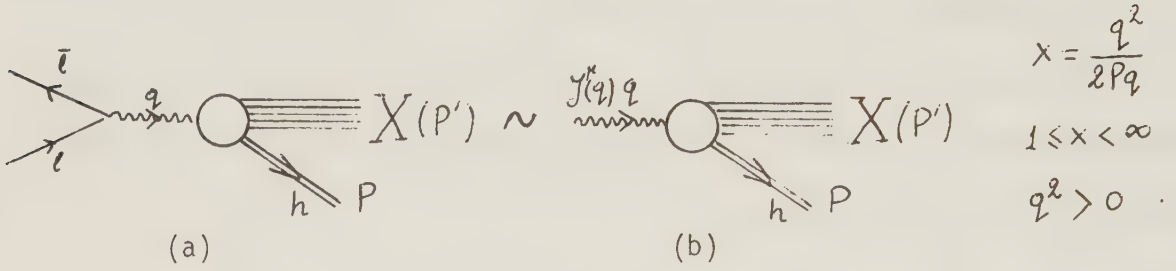
avec

$$x = q^2/2Pq \;, \quad q^2 < 0 \quad \text{et} \quad 0 \leq x \leq 1 \qquad (2b)$$

ii) Annihilation semi-inclusive

Nous considérons l'annihilation semi-inclusive d'un lepton avec son antilepton donnant dans l'état final un hadron h (P) déterminé

l (lepton) + l (antilepton) ⟶ h (hadron) + X, (fig.2a).



(a) (b)

Fig. 2 : Annihilation semi-inclusive. (a) l + l → h + X (b) $J \to h + X$

Comme dans la diffusion, nous négligeons les puissances supérieures du couplage électromagnétique (ou faible); il suffit donc d'examiner la production d'un hadron h à partir d'un courant $J^\mu(q)$:

$$J(q) \longrightarrow X(P') + h(P) \tag{3}$$

Les variables cinématiques sont définies dans la figure (2b).

La fonction de fragmentation $W^{\mu\nu}$ est alors définie comme suit:

$$\tilde{W}^{\mu\nu}(x, q^2, P^2) \equiv \sum_X \int d^4y\, e^{-iqy} \langle 0|J^\nu(0)|X;P\rangle\langle X;P|J^{+\mu}(y)|0\rangle \tag{4a}$$

avec $x = q^2/2Pq$, $q^2 > 0$ et $1 \leq x < \infty$ (4b).

Grâce à l'invariance de jauge, $W^{\mu\nu}$ et $\tilde{W}^{\mu\nu}$ peuvent être décomposées en trois fonctions de structure scalaires F_2 , F_L et F_3 et en trois fonctions de fragmentation scalaires $\tilde{F}_2$, $\tilde{F}_L$ et $\tilde{F}_3$.

$$\overset{(\sim)}{W}{}^{\mu\nu}(x, q^2, P^2) = \left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) \overset{(\sim)}{F}_L(x, q^2, P^2) + i\varepsilon^{\mu\nu\rho\sigma} \frac{P_\rho q_\sigma}{Pq} \overset{(\sim)}{F}_3(x, q^2, P^2)$$
$$\left(\frac{P^\mu P^\nu}{Pq} - \frac{P^\mu q^\nu + P^\nu q^\mu}{q^2} - \frac{Pq}{q^2} g^{\mu\nu}\right) \overset{(\sim)}{F}_2(x, q^2, P^2) \tag{5}$$

Pour le courant électromagnétique, $\overset{(\sim)}{F}_3$ est égale à zero (par suite de la conservation de la parité) mais pas pour le courant faible.

Expérimentalement les fonctions de structure, F_2 , F_L et F_3 sont assez bien connues jusqu'à $-q^2 = 250$ Gev^2 pour une région de x comprise entre 0.03 et 0.9 [1] ; au contraire $\tilde{F}_2$, $\tilde{F}_L$ et $\tilde{F}_3$ sont mal connues actuellement. [2]

2. Le comportement asymptotique des fonctions de structure et de fragmentation.

Dans le chapitre suivant nous allons démontrer que les fonctions de structure et les fonctions de fragmentation se décomposent en sommes de termes factorisés quand $|q^2| \to \infty$. Si $F(x,q^2,p^2)$ réprésente une fonction de structure et $\tilde{F}(x,q^2,p^2)$ une fonction de fragmentation, on a les expressions suivantes valables dans la limite de grand $|q^2|$: [3]

$$\int_0^1 dx\, x^{\sigma-1} F(x,q^2,p^2) \xrightarrow[-q^2\to\infty]{} \sum_i C_\sigma^i(q^2/\mu^2)\, \Gamma_\sigma^i(p^2/\mu^2) + \Theta(1/q^2) \quad (6a)$$

$$\int_1^\infty dx\, x^{-\sigma-1} \tilde{F}(x,q^2,p^2) \xrightarrow[q^2\to\infty]{} \sum_i \tilde{C}_\sigma^i(q^2/\mu^2)\, \Gamma_\sigma^i(p^2/\mu^2) + \Theta(1/q^2) \quad (6b)$$

où $\Gamma_\sigma^i(p^2/\mu^2)$ et $\Gamma_\sigma^i(p^2/\mu^2)$ sont des fonctions qui ne dépendent pas du moment de transfert q^2 et qu'on appelle des vertex coupés du genre espace (genre temps). Ils dépendent seulement du carré du quadrimoment p^μ du hadron. Les $\overset{(\sim)}{C}_\sigma^i(q^2/\mu^2)$ dépendent seulement du grand moment de transfert q^2 et on les appelle "fonctions coefficients" des vertex-coupés correspondants. L'indice i correspond aux quarks, antiquarks ou gluons. Enfin, l'échelle μ^2 s'introduit par la renormalisation des vertex coupés. Les définitions des vertex coupés sont données dans le paragraphe 3 ci-dessous.

3. La notion des vertex coupés

Les équations (6 a et 6 b) sont très importantes. Grâce à la propriété

de liberté asymptotique [4] la partie qui dépend du grand moment de transfert q^2 est calculable en fonction de la constante de couplage effective $\alpha(q^2)$ qui a la propriété de décroître avec $|q^2|$.

D'autre part les vertex coupés $\tilde{\Gamma}^{(\sim)i}_{\sigma}(P^2/\mu^2)$ ont une interprétation probabiliste très importante que nous allons essayer d'expliquer dans la suite.

Pour simplifier les notations, plaçons nous pour l'instant dans une théorie de champ scalaire neutre en interaction donnée par un terme en ϕ^3 dans un espace-temps à six dimensions. Cette théorie de champ partage la propriété de liberté asymptotique avec la chromodynamique quantique mais, elle est beaucoup plus simple. Elle contient seulement deux types de vertex coupés, un du genre espace $\Gamma_{\sigma}(P^2/\mu^2)$ et un du genre temps $\tilde{\Gamma}_{\sigma}(P^2/\mu^2)$. Γ_{σ} et $\tilde{\Gamma}_{\sigma}$ sont définis à partir des expressions suivantes :

$$\Gamma_{\sigma}(P^2/\mu^2) = 1 + \int \frac{d^6K}{(2\pi)^6} \left| A(K,P)G(K) \right|^2 \cdot \left(\frac{\Delta K}{\Delta P}\right)^{\sigma} \cdot \theta\left(\frac{\Delta K}{\Delta P}\right) \qquad (7a)$$

$$\tilde{\Gamma}_{\sigma}(P^2/\mu^2) = 1 + \int \frac{d^6K}{(2\pi)^6} \left| \tilde{A}(K,P)G(K) \right|^2 \cdot \left(\frac{\Delta K}{\Delta P}\right)^{-\sigma} \theta\left(\frac{\Delta K}{\Delta P}\right) \qquad (7b)$$

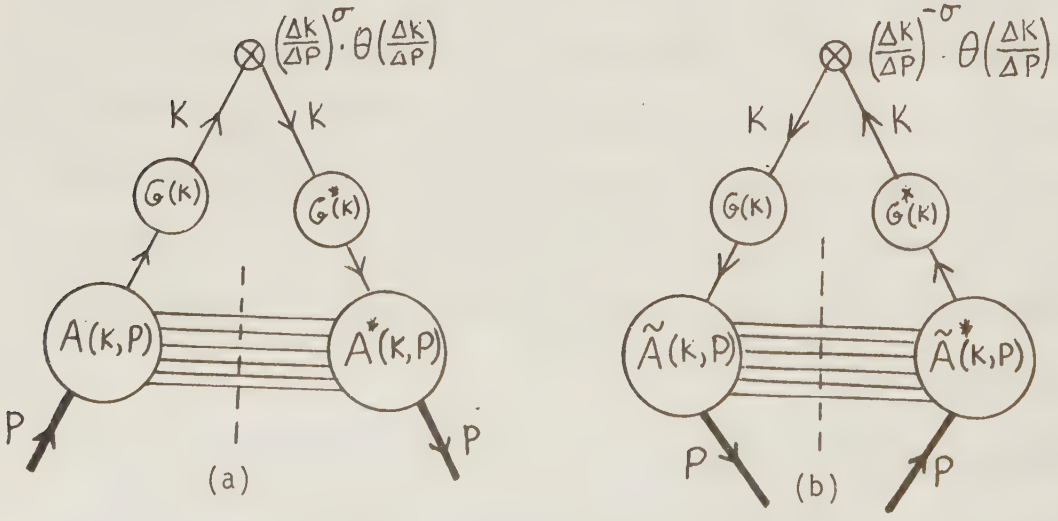


Fig. 3 : Vertex coupés : (a) du genre espace (b) du genre temps

$G(K)$, $A(K,P)$ et $\tilde{A}(K,P)$ sont des fonctions de Green définies dans les figures (3a) et (3b). Le vecteur Δ est du genre lumière $\Delta^2 = 0$; et antiparallèle à P.

$$P = (P_0, P_3, \vec{0}) \quad , \quad \Delta = (1, -1, \vec{0}) \quad , \quad \Delta P = (P_0 + P_3) \equiv P^+ > 0 \qquad (8)$$

Le noyau $|A(K,P)\,G(K)|^2$ réprésente la probabilité de production d'un état virtuel $\varphi(K)$ à partir d'un état asymptotique $|P\rangle$.

De façon symétrique, le produit $|\tilde{A}(K,P)G(K)|^2$ représente la probabilité de production d'un état asymptotique $|P\rangle$ à partir d'un état virtuel $\varphi(K)$.

$$|A(K,P)G(K)|^2 = \sum_X |\langle P \mid \varphi(K) \mid X\rangle|^2 \qquad (9a)$$

et

$$|\tilde{A}(K,P)G(K)|^2 = \sum_X |\langle 0 \mid \varphi(K) \mid X;P\rangle|^2 \qquad (9b)$$

Il en résulte que les vertex coupés sont le résultat d'une intégration d'un noyau probabiliste $\overset{(\sim)}{P}(K,P)$ avec un poids $\left(\frac{\Delta K}{\Delta P}\right)^{(-)\sigma} \cdot \theta\left(\frac{\Delta K}{\Delta P}\right)$.

$$\Gamma_\sigma(P^2/\mu^2) = \int \frac{d^6K}{(2\pi)^6}\, P(K,P) \cdot \left(\frac{\Delta K}{\Delta P}\right)^{\sigma} \cdot \theta\left(\frac{\Delta K}{\Delta P}\right) \qquad (10a)$$

et

$$\tilde{\Gamma}_\sigma(P^2/\mu^2) = \int \frac{d^6K}{(2\pi)^6}\, \tilde{P}(K,P) \cdot \left(\frac{\Delta K}{\Delta P}\right)^{-\sigma} \cdot \theta\left(\frac{\Delta K}{\Delta P}\right) \qquad (10b)$$

avec

$$\overset{(\sim)}{P}(K,P) = \left[(2\pi)^6 \delta^6(K-P) + \left|\overset{(\sim)}{A}(K,P)\, G(K)\right|^2\right] \theta\left(\frac{\Delta K}{\Delta P}\right) \qquad (11)$$

ou bien

$$\Gamma_\sigma(P^2/\mu^2) = \int_0^1 dx\, x^{\sigma-1} \int \frac{d^6K}{(2\pi)^6}\, x\, P(K,P)\, \delta\left(x - \frac{\Delta K}{\Delta P}\right) \qquad (12a)$$

$$\tilde{\Gamma}_\sigma(P^2/\mu^2) = \int_1^\infty dx\, x^{-\sigma-1} \int \frac{d^6k}{(2\pi)^6}\, x\, \tilde{\mathcal{P}}(k,P)\, \delta\left(x - \frac{\Delta K}{\Delta P}\right) \qquad (12b)^{(+)}$$

$$\Gamma_\sigma(P^2/\mu^2) = \int_0^1 dx\, x^{\sigma-1}\, q(x,\mu) \qquad (13a)$$

$$\tilde{\Gamma}_\sigma(P^2/\mu^2) = \int_1^\infty dx\, x^{-\sigma-1}\, \tilde{q}(x,\mu) \qquad (13b)$$

Dans ces formules,

$$q(x,\mu) = \int \frac{d^6k}{(2\pi)^6}\, x\, \mathcal{P}(k,P)\, \delta\left(x - \frac{\Delta K}{\Delta P}\right) \cdot \theta(1-x)\,\theta(x) \qquad (14a)$$

représente la probabilité de production, à partir de l'état $|P\rangle$, d'un état virtuel $\varphi(K)$, ayant une fraction du moment longitudinal x : $0 \leq x = \frac{\Delta K}{\Delta P} \leq 1$

et $$\tilde{q}(x,\mu) = \int \frac{d^6k}{(2\pi)^6}\, x\, \tilde{\mathcal{P}}(k,P)\, \delta\left(x - \frac{\Delta K}{\Delta P}\right) \cdot \theta(x-1) \qquad (14b)$$

représente la probabilité de production d'un état $|P\rangle$ à partir d'un état virtuel $\varphi(k)$ ayant une fraction de moment X : $X = \frac{\Delta K}{\Delta P} \geq 1$

On observe alors que Γ_σ n'est rien d'autre que la transformée de Mellin de la fonction de distribution de partons $q(x,\mu)$ et $\tilde{\Gamma}_\sigma$ la transformée de Mellin de la fonction de fragmentation de partons $\tilde{q}(x,\mu)$.

L'échelle μ est le point où Γ_σ et $\tilde{\Gamma}_\sigma$ sont rénormalisées. En effet, les vertex coupés ont des divergences ultraviolettes. On les rend finies en choisissant

---

(+): La contrainte $0 \leq x = \frac{\Delta K}{\Delta P} \leq 1$ dans le cas de Γ_σ vient d'abord de la fonction $\theta\left(\frac{\Delta K}{\Delta P}\right)$ qui apparaît dans la définition Γ_σ et ensuite du fait que $P'^+ = P^+ - K^+$ est positif (production des particules sur leur couche de masse). La contrainte $X \geq 1$ dans le cas de $\tilde{\Gamma}_\sigma$ vient du fait que $P'^+ = K^+ - P^+$ est positif.

des conditions de renormalisation, ce qui introduit l'échelle μ .

Dans le cas de la chromodynamique quantique il existe plusieurs vertex coupés correspondant aux différents champs des quarks, antiquarks ou des gluons.

L'image des vertex coupés donnée dans le cas des fonctions de distribution (et de fragmentation) de partons scalaires, ainsi que la transformation de Mellin, restent valables dans la chromodynamique quantique avec une complication provenant de l'invariance de jauge.

Dans ce cas, en effet, l'état virtuel d'un quark par exemple, n'est plus bien défini car en effectuant une transformation de jauge le quark se transforme en un état cohérent d'un quark plus un nombre indéterminé de gluons longitudinaux. [5]

En réalité le vertex coupé d'un quark (gluon) de la chromodynamique quantique est un objet invariant de jauge et il exprime la transformée de Mellin de la probabilité de production d'un état cohérent de quark (gluon) + un nombre indéterminé des gluons longitudinaux, à partir d'un état physique (invariant de jauge) $|P\rangle$. Le choix de l'état cohérent est tel qu'il estinvariant par les transformations de jauge (voir figures 4a,5a).

Dans la jauge particulière $\Delta A = A^{+} = 0$, on retrouve, pour les théories de jauge, l'image simplifiée d'une théorie scalaire [6] , voir figures (4,5).

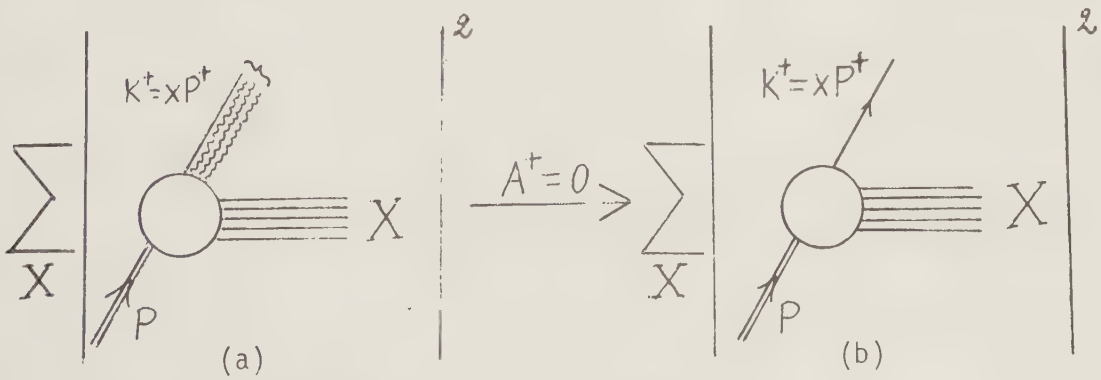
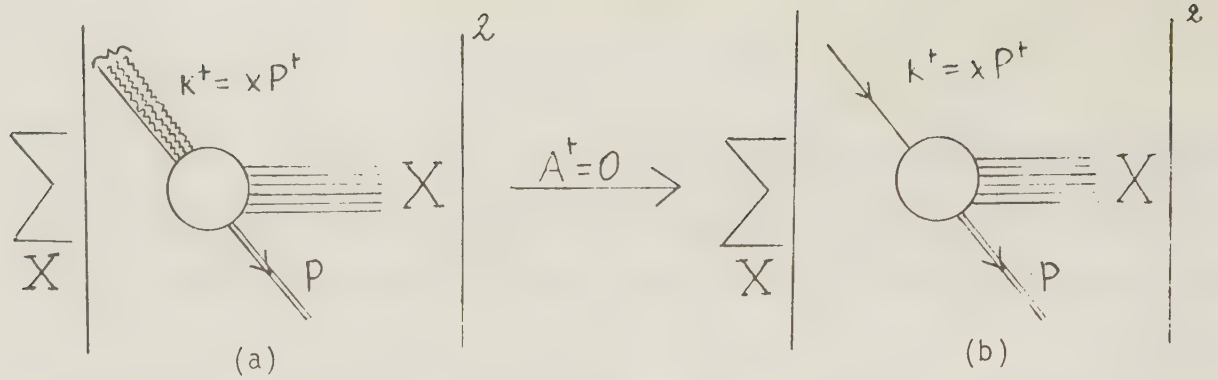


Fig. 4 : L'image probabiliste des vertex coupés du genre espace.
(a) dans une jauge générale (b) dans la jauge $\Delta A = A^{+} = 0$



(a) (b)

Fig. 5 : L'image probabiliste des vertex coupés du genre temps.
(a) dans une jauge générale (b) dans la jauge $\Delta A = A^+ = 0$

On peut alors récrire l'équation (6a) à l'aide de fonctions de distribution de quarks $q(x,\mu)$, antiquarks $\bar{q}(x,\mu)$ et gluons $G(x,\mu)$, et l'équation (6b) à l'aide de la probabilité de fragmentation en hadrons d'un quark $\tilde{q}(x,\mu)$, d'un antiquark $\tilde{\bar{q}}(x,\mu)$ et d'un gluon $\tilde{G}(x,\mu)$.

$$F(x,q^2,P^2) = \sum_i C_i\left(x,\frac{-q^2}{\mu^2}\right) \otimes q_i(x,\mu) \qquad (15a)$$

i = quark, antiquark ou gluon

$$\tilde{F}(x,q^2,P^2) = \sum_i \tilde{C}_i\left(x,\frac{q^2}{\mu^2}\right) \otimes \tilde{q}_i(x,\mu) \qquad (15b)$$

i = quark, antiquark ou gluon

où le produit de convolution $\otimes$ entre deux fonctions est défini comme :

$$A(x) \otimes B(x) = \int_x^1 \frac{dy}{y} A\left(\frac{x}{y}\right) B(y) \qquad (16a)$$

dans le cas de la diffusion $(0 \leq X \leq 1)$ et

$$\tilde{A}(x) \otimes \tilde{B}(x) = \int_1^x \frac{dy}{y} A\left(\frac{x}{y}\right) B(y) \qquad (16b)$$

dans le cas de l'annihilation $(1 \leq X < \infty)$.

$\overset{(\sim)}{C}_i\left(x,(\mp)\,q^2/\mu^2\right)$ sont les transformées de Mellin inverses des fonctions coefficients $\overset{(\sim)}{C}_i^{\sigma}\left(q^2/\mu^2\right)$.

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- III -

LE MODELE DES PARTONS GENERALISE

Dans ce chapitre, nous introduisons le modèle des partons généralisé, qui permet d'exprimer les fonctions de structure et les fonctions de fragmentation d'une manière probabiliste.

Fonctions de structure.

Il est clair que la diffusion profondément inélastique est un processus qui permet l'étude de la structure hadronique.

Si l'on oublie pour l'instant les interactions fortes ($\alpha(0) = 0$), le modèle naïf des partons [1] permet de paramétriser les fonctions de structure en introduisant les distributions de partons dans un hadron, $q_i^h(x)$. Par exemple, dans le cas de la réaction $e^- p \to e^- x$ on trouve pour la fonction de structure $F_2^{ep}(x,Q)$ l'équation suivante : (voir figure 1a)

$$F(x,Q) = \frac{1}{x} F_2^{ep}(x,Q) = \sum_i \int_x^1 \frac{dz}{z} e_i^2 \delta(1-\tfrac{x}{z})\, q_i^p(z) \qquad (1)$$

Dans cette expression, e_i est la charge électromagnétique du quark (ou de l'antiquark) i , z est la fraction de l'impulsion longitudinale portée par le parton, $z = k^+/p^+$, et x est la variable de Feynman $x = (q^2/2pq) = q^+/p^+$
On a choisi le référentiel où :

$$q^\mu = (q^o, q^3, \vec{o}) \;, \quad p^\mu = (p^o, p^3, \vec{o}) \;; \quad Q \equiv \sqrt{-q^2} \qquad (2)$$

Dans le chapitre II, eq. (15a), on a remarqué que les fonctions de structure s'expriment en termes des fonctions de distribution des quarks, antiquarks et gluons $q_i(x,\mu)$.
En choisissant l'échelle de renormalisation $\mu^2 = Q^2 \equiv -q^2$ dans $F(x,Q)$ les prédictions des vertex coupés s'expriment comme suit (voir figure 1b)

$$F(x,Q) = \sum_i \int_x^1 \frac{dz}{z} E_i^p\left(\frac{x}{z}, \alpha(Q)\right) q_i^p(z,Q) \qquad (3)$$

où $E_i^p(x,\alpha(Q))$ est la transformée de Mellin inverse de la fonction coefficient $C_\sigma^i(-q^2/\mu^2, \alpha_\mu)$ (eq. II.6a) pour $|q^2| \equiv Q^2 = \mu^2$

avec de plus

$$Q\frac{\partial}{\partial Q} q_i^p(x,Q) = \sum_i \int_x^1 \frac{dy}{y} P_{ij}\left(\frac{x}{y}, \alpha(Q)\right) q_i^p(y,Q) \qquad (4)$$

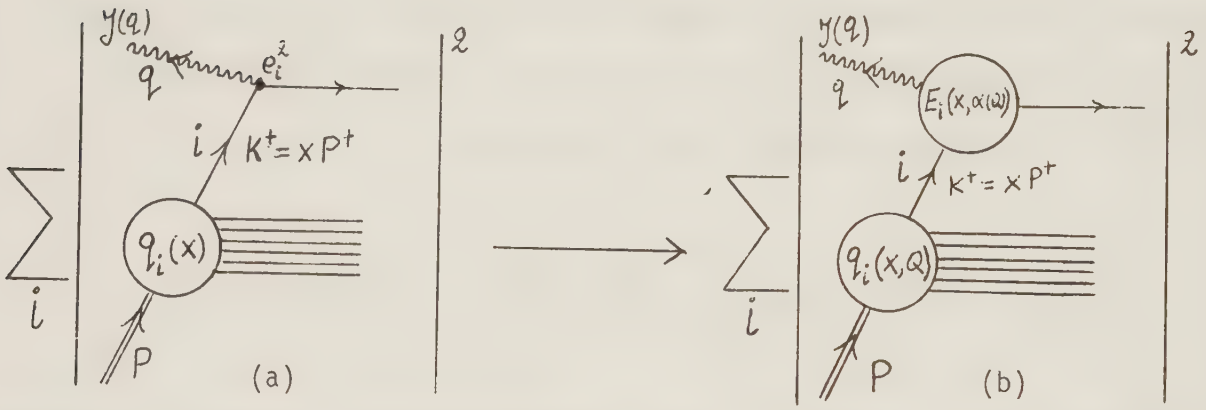

Fig. 1 : La diffusion inélastique vue par : (a) le modèle des partons naïf, (b) le modèle des partons généralisé

Cette dernière équation est une conséquence de la renormalisabilité des vertex coupés [4]. Les noyaux d'évolution $P_{ij}(x,\alpha(Q))$ sont reliés aux dimensions anormales des vertex coupés $\gamma_{ij}^\sigma(\alpha)$ [2,3].

$$\int_0^1 dx\, x^{\sigma-1} P_{ij}(x,\alpha) = -\gamma_{ij}^\sigma(\alpha) \qquad ij = \text{quark, antiquark, gluon.} \qquad (5)$$

Le formalisme des vertex coupés conduit alors à une généralisation du modèle des partons naïf. Les distributions des partons ont une dépendance dans la variable Q déterminée par l'équation (4). De plus, l'interaction ponctuelle $e_i^2 \delta(1 - x/z)$ a été remplacée par une fonction qui dépend de $\alpha(Q)$, $E_i^p(x/z, \alpha(Q))$ (figs. 1a, 1b).

Les quantités qui sont calculables en théorie des perturbations sont précisément les noyaux $P_{ij}(x, \alpha(Q))$ et $E_i^P(x, \alpha(Q))$. Elles s'expriment en séries de puissances de la constante de couplage effective $\alpha(Q)$ qui diminue avec Q grâce à la liberté asymptotique. Si Q est suffisamment grand, il suffit de considérer quelques puissances de $\alpha(Q)$

$$P_{ij}(x, \alpha(Q)) = \frac{\alpha(Q)}{4\pi} P_{ij}^0(x) + \left(\frac{\alpha(Q)}{4\pi}\right)^2 P_{ij}^1(x) + \ldots \qquad (6a)$$

et $$E_i^P(x, \alpha(Q)) = e_i^2 \delta(1-x) + \frac{\alpha(Q)}{4\pi} E_i^1(x) + \ldots \qquad (6b)$$

On remarque que le modèle naïf des partons n'est autre que l'approximation où $\alpha(Q) \simeq 0$. On comprend alors que ce modèle soit relativement satisfaisant, étant donné que les corrections $\alpha(Q)$ sont de l'ordre de 20 %, ce qui, dans le passé, était compatible avec les erreurs statistiques sur les fonctions de structure expérimentales. En revanche, la statistique est aujourd'hui bien meilleure [5], ce qui permet d'étudier les effets d'ordre $\alpha(Q)$ et même $\alpha^2(Q)$.

Fonctions de fragmentation

Si on suit le même schéma que précédemment, on découvre une analogie entre la diffusion profondément inélastique et l'annihilation semi-inclusive. Comme dans le cas de la fonction de structure, si $\alpha(Q) \equiv 0$, le modèle naïf des partons [1] permet d'exprimer les fonctions de fragmentation à l'aide de quantités dites "fonctions de fragmentation des partons en hadrons", que l'on symbolise par $\tilde{q}_i^h(x)$, la variable X est ici plus grande que l'unité ($X \geqslant 1$). Une fonction de fragmentation par exemple $\tilde{F}_2^{e^+e^- \to \pi}(x, Q)$, pourra s'écrire comme suit : (fig. 2a)

$$\tilde{F}(x, Q) = \frac{1}{x} \tilde{F}_2^{e^+e^- \to \pi}(x, Q) = \sum_i \int_x^X \frac{dz}{z} e_i^2 \delta\left(1 - \frac{x}{z}\right) \tilde{q}_i^\pi(z) \qquad (7)$$

Le formalisme des vertex coupés du genre temps prédit que [3]

(fig. 2b)

$$\tilde{F}(x,Q) = \sum_i \int_x^1 \frac{dz}{z}\, \tilde{E}_i^n\left(\frac{x}{z}, \alpha(Q)\right)\, \tilde{q}_i^n(z,Q) \qquad (8a)$$

avec

$$Q\frac{\partial}{\partial Q}\, \tilde{q}_i^n(x,Q) = \int_x^1 \frac{dz}{z}\, \tilde{P}_{ij}\left(\frac{x}{z}, \alpha(Q)\right)\, \tilde{q}_i^n(z,Q) \qquad (8b)$$

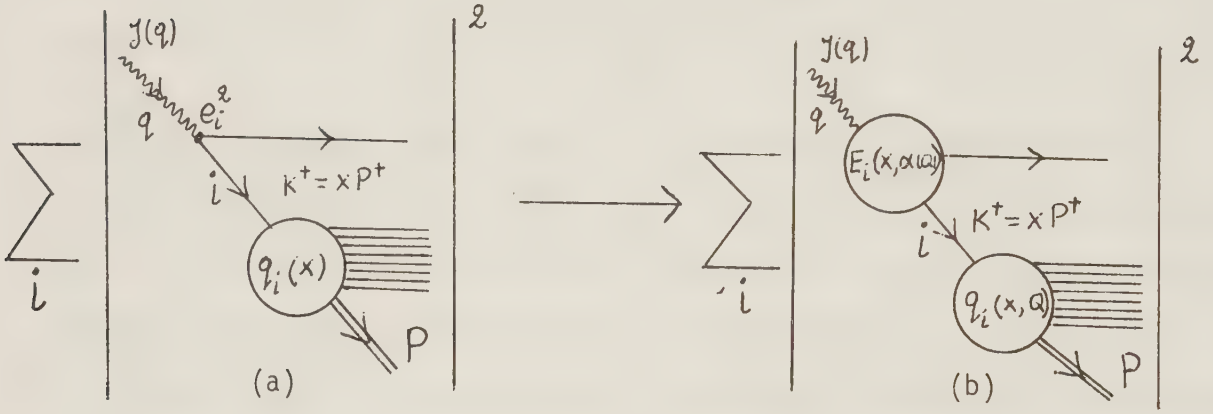

Fig. 2 : l'annihilation semi-inclusive vue par : (a) le modèle des partons naïf (b) le modèle des partons généralisé

Comme dans le cas des fonctions de structure, on trouve, ici aussi, que le modèle naïf des partons coïncide avec l'approximation $\alpha(Q) = 0$ du modèle des partons généralisé.

Les charges effectives $\tilde{E}_i^n(x,\alpha(Q))$ sont reliées aux fonctions coefficients $\tilde{C}_\sigma^i\left(q^2/\mu^2, \alpha_\mu\right)$:

$$\tilde{C}_\sigma^i\,(1,\alpha(Q)) = \int_1^\infty dx\; x^{-\sigma-1}\, \tilde{E}_i^n(x,\alpha(Q)) \qquad (9)$$

tandis que les dimensions anormales $-\tilde{\gamma}_{ij}^\sigma(\alpha)$ sont les transformées de Mellin du noyau d'évolution $\tilde{P}_{ij}(x,\alpha)$ [3]

$$\int_1^\infty dx\; x^{-\sigma-1}\, \tilde{P}_{ij}(x,\alpha) = -\gamma_{ij}^\sigma(\alpha) \qquad (10)$$

Toutefois, les fonctions de fragmentation ne sont pas aussi bien mesurées que les fonctions de structure. Néanmoins, il est clair qu'il existe une violation significative de l'invariance d'échelle, qui, au moins qualitativement, est en accord avec les prédictions de la chromodynamique quantique. Un effet particulier aux fonctions de fragmentation est que, pour $1/x < 0.2$, on observe une violation de l'invariance d'échelle beaucoup plus importante que pour $1/x > 0.2$, où $\tilde{F}(x,Q)$ ne dépend pratiquement pas de Q. [6] Cela peut s'expliquer par des effets d'ordre α^2 pour lesquels les corrections sont relativement grandes pour les petites valeurs de $1/x$. [7]

Relation entre F et $\tilde{F}$.

Puisqu'il existe des prédictions précises, à la fois pour les fonctions de structure (F) et de fragmentation ($\tilde{F}$), il est important d'examiner si ces fonctions sont reliées. Plus précisément, nous cherchons à trouver s'il existe des relations entre les quantités calculables P_{ij} et $\tilde{P}_{ij}$, ainsi qu'entre E_i et $\tilde{E}_i$. La première relation examinée est la relation de réciprocité de Gribov et Lipatov: [8]

$$A_{ij}(x) = x^{d_A} \tilde{A}_{ji}\left(\frac{1}{x}\right) \qquad (11a)$$

ou bien (+)

$$\langle A_{ij} \rangle_\sigma = \langle \tilde{A}_{ij} \rangle_{\sigma + d_A} \qquad (11b)$$

où A(x) est une quantité de diffusion et $\tilde{A}(x)$ une quantité d'annihilation.

$\langle A \rangle_\sigma$ exprime la transformée de Mellin de la fonction A(x).

Cette relation est valable pour toutes les théories des champs à

(+) On utilise dans la suite la notation $\langle A \rangle_\sigma$ pour représenter l'intégrale $\int_0^1 dx\, x^{\sigma-1} A(x)$ dans le cas de diffusion et $\int_1^\infty dx\, x^{-\sigma-1} A(x)$ dans le cas d'annihilation.

l'approximation d'ordre $\alpha(Q)$, et en particulier pour la chromodynamique quantique. On verra, dans les chapitres suivants, que cette relation est violée, au niveau des diagrammes à deux boucles, pour la théorie scalaire [3,9] $[\phi^3]_6$ comme pour la chromodynamique quantique [7,10,11].

Une autre relation valable au niveau des diagrammes à une boucle est celle de la continuation analytique [12] :

$$A(x) = - Re\, \tilde{A}(x) \quad , \quad x<1 \tag{12a}$$

$Re\, \tilde{A}(x)$ est la partie réelle du prolongement analytique de $\tilde{A}(x)$ dans la région non physique $x<1$. Pour la transformée de Mellin on a

$$\langle A \rangle_\sigma = Re \langle \tilde{A} \rangle_{-\sigma} \quad , \quad Re\,\sigma > 0 \tag{12b}$$

Cette dernière relation, telle qu'elle est écrite, n'est pas valable pour la chromodynamique quantique, même à l'ordre $\alpha(Q)$, puisque dans l'expression en σ de $\langle A \rangle_\sigma$ et $\langle \tilde{A} \rangle_\sigma$ apparaissent des sommes du type:

$$S(\sigma) = \sum_{j=1}^{\sigma} \frac{1}{j} = \psi(\sigma+1) - \psi(1) \tag{13}$$

qui ne sont pas définies pour σ = entier négatif.

Nous démontrerons l'existence d'une relation plus générale, qui définit bien la continuation de la distribution en x, et en même temps l'opération $\sigma \to -\sigma$. Nous avons trouvé que cette relation reste valable à l'ordre $\alpha^2(Q)$ pour la théorie $[\phi^3]_6$ [3,9] comme pour la chromodynamique quantique. [7,10]

Il existe encore une autre relation entre les différents éléments de la matrice P_{ij} et $\tilde{P}_{ij}$, que l'on appelle "relation supersymétrique"

$$\left[P_{qq}(x,\alpha) + P_{Gq}(x,\alpha) = P_{qG}(x,\alpha) + P_{GG}(x,\alpha) \right]_{C_A = C_F = 2T_R} \tag{14a}$$

$$\left[\tilde{P}_{qq}(x,\alpha) + \tilde{P}_{gq}(x,\alpha) = \tilde{P}_{qg}(x,\alpha) + \tilde{P}_{gg}(x,\alpha)\right]_{C_A = C_F = 2T_R} \qquad (14b)$$

Dans le cas où les gluons et les quarks se placent dans des représentations de l'algèbre de supersymétrie ($C_A = C_F = 2\ T_R$), et si la renormalisation s'effectue sans briser cette invariance SS , les relations (14a,b) font que les éléments de matrice des noyaux d'évolution entre les quarks et les gluons ne sont pas des quantités indépendantes. [7,14]

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- IV -

DEMONSTRATION DE LA FACTORISATION DANS LA REGION DES GRANDS MOMENTS DE TRANSFERT

En utilisant le formalisme des vertex coupés introduit par Mueller, j'ai établi en collaboration avec Antoniadis et Baulieu (Nucl.Phys. B 168 (1980) 394), une démonstration simple du théorème de factorisation dans les processus d'électroproduction et d'annihilation semi-inclusive. Dans ce chapitre, nous expliquons cette démonstration en nous limitant, pour plus de simplicité, au cas de l'électroproduction.

Cette méthode de démonstration relie d'une manière claire le comportement asymptotique des fonctions de structure (fragmentation), à une propriété beaucoup plus générale, celle de la renormalisabilité de la théorie des champs. Etant donné que les propriétés de factorisation, dans la région asymptotique, ont une interprétation probabiliste, le modèle des partons généralisé apparaît alors comme une conséquence simple de la renormalisabilité de la théorie.

Soit $F_{\gamma q}(\frac{p^2}{\mu^2}, \frac{q^2}{\mu^2}, \alpha_\mu)$ la fonction de structure de la diffusion du courant électromagnétique par le quark et $F_{\gamma G}(\frac{p^2}{\mu^2}, \frac{q^2}{\mu^2}, \alpha_\mu)$ la fonction de structure de la diffusion du courant électromagnétique par le gluon. Les équations différentielles suivantes découlent de la renormalisabilité [1] :

$$D\, F_{\gamma q}(p^2/\mu^2, q^2/\mu^2, \alpha_\mu) = 0 \tag{1a}$$

$$D\, F_{\gamma G}(p^2/\mu^2, q^2/\mu^2, \alpha_\mu) = 0 \tag{1b}$$

où D est l'opérateur différentiel du groupe de renormalisation:

$$D = \mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha) \frac{\partial}{\partial \alpha} + \gamma_m(\alpha)\, m^2 \frac{\partial}{\partial m^2} \tag{2}$$

Dans ce qui suit, nous négligeons toujours le terme $\gamma_m(x)\, m^2 \frac{\partial}{\partial m^2}$ pour simplifier les calculs algébriques, ce qui n'a pas de conséquences essentielles pour les résultats finals.

Il découle également de la renormalisabilité que les vertex coupés d'un photon (qui ont comme noyaux $F_{\gamma q}$ et $F_{\gamma G}$) sont aussi renormalisables. [1] Nous désignons par $\Gamma^{\sigma}_{\gamma q}$ et $\Gamma^{\sigma}_{\gamma G}$ les vertex coupés correspondant à $F_{\gamma q}$ et $F_{\gamma G}$:

$$\Gamma^{\sigma}_{\gamma q}(P^2/\mu^2) = 1 + \int \frac{d^4K}{K^4} \frac{1}{(2\pi)^4} F_{\gamma q}(K,P) \left(\frac{\Delta K}{\Delta P}\right)^{\sigma} \theta\left(\frac{\Delta K}{\Delta P}\right) \qquad (3a)$$

et
$$\Gamma^{\sigma}_{\gamma G}(P^2/\mu^2) = 1 + \int \frac{d^4K}{K^4} \frac{1}{(2\pi)^4} F_{\gamma G}(K,P) \left(\frac{\Delta K}{\Delta P}\right)^{\sigma} \theta\left(\frac{\Delta K}{\Delta P}\right) \qquad (3b).$$

D'autre part, désignons par Γ^{σ}_{qq} et Γ^{σ}_{qG} les éléments de matrice des vertex coupés d'un quark dans les états à un quark et un gluon, et Γ^{σ}_{Gq} et Γ^{σ}_{GG} les vertex coupés d'un gluon dans les états à un quark et un gluon. Comme $\Gamma^{\sigma}_{\gamma q}$, Γ^{σ}_{qq}, Γ^{σ}_{Gq} ont les mêmes nombres quantiques, ainsi que $\Gamma^{\sigma}_{\gamma G}$, Γ^{σ}_{qG}, Γ^{σ}_{GG}, ils se mélangent par renormalisation. D'où l'équation suivante:

$$\Gamma^{\sigma}_{Ren.} \equiv \begin{bmatrix} 1 & \Gamma^{\sigma}_{\gamma q} & \Gamma^{\sigma}_{\gamma G} \\ 0 & \Gamma^{\sigma}_{qq} & \Gamma^{\sigma}_{qG} \\ 0 & \Gamma^{\sigma}_{Gq} & \Gamma^{\sigma}_{GG} \end{bmatrix}_{Ren} = \begin{bmatrix} 1 & Z^{\sigma}_{\gamma q} & Z^{\sigma}_{\gamma G} \\ 0 & Z^{\sigma}_{qq} & Z^{\sigma}_{qG} \\ 0 & Z^{\sigma}_{Gq} & Z^{\sigma}_{GG} \end{bmatrix} \cdot \begin{bmatrix} 1 & \Gamma^{\sigma}_{\gamma q} & \Gamma^{\sigma}_{\gamma G} \\ 0 & \Gamma^{\sigma}_{qq} & \Gamma^{\sigma}_{qG} \\ 0 & \Gamma^{\sigma}_{Gq} & \Gamma^{\sigma}_{GG} \end{bmatrix}_{Nu} \qquad (4)$$

Comme les contretermes Z^{σ}_{ij} ne dépendent pas de P^2 (localité des contretermes), on peut remplacer, dans l'équation (4), Γ^{σ}_{ij} par sa dérivée $\dot{\Gamma}^{\sigma}_{ij} = P^2 \frac{\partial}{\partial P^2} \Gamma^{\sigma}_{ij}$. Cela nous permet, en éliminant les Z^{σ}_{ij}, de relier $(\Gamma^{\sigma}_{ij})_{Nu}$ à sa dérivée $P^2 \frac{\partial}{\partial P^2} (\Gamma^{\sigma}_{ij})_{Nu}$.

$$\left[\begin{pmatrix} P^2 \frac{\partial}{\partial P^2} & 0 \\ 0 & P^2 \frac{\partial}{\partial P^2} \end{pmatrix} - \begin{pmatrix} \mathcal{D}^{\sigma}_{qq} & \mathcal{D}^{\sigma}_{qG} \\ \mathcal{D}^{\sigma}_{Gq} & \mathcal{D}^{\sigma}_{GG} \end{pmatrix} \right] \cdot \begin{pmatrix} \Gamma^{\sigma}_{\gamma q} \\ \Gamma^{\sigma}_{\gamma G} \end{pmatrix} = \begin{pmatrix} \mathcal{D}^{\sigma}_{\gamma q} \\ \mathcal{D}^{\sigma}_{\gamma G} \end{pmatrix} \qquad (5)$$

Les $\mathcal{D}_{ij}^{\sigma}$ sont des quantités finies déterminées par l'équation

$$\begin{bmatrix} 1 & \mathcal{D}_{\gamma q}^{\sigma} & \mathcal{D}_{\gamma G}^{\sigma} \\ 0 & \mathcal{D}_{qq}^{\sigma} & \mathcal{D}_{qG}^{\sigma} \\ 0 & \mathcal{D}_{Gq}^{\sigma} & \mathcal{D}_{GG}^{\sigma} \end{bmatrix} = \Gamma_{Ren}^{\sigma\,-1} \cdot \dot{\Gamma}_{Ren}^{\sigma} \tag{6}$$

En choisissant, par exemple, les conditions de renormalisation

$$\begin{pmatrix} \Gamma_{qq}^{\sigma} & \Gamma_{qG}^{\sigma} \\ \Gamma_{Gq}^{\sigma} & \Gamma_{GG}^{\sigma} \end{pmatrix}_{p^2=-\mu^2} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{7}$$

on observe que les quantités $\mathcal{D}_{qq}^{\sigma}$, $\mathcal{D}_{Gq}^{\sigma}$, $\mathcal{D}_{qG}^{\sigma}$, $\mathcal{D}_{GG}^{\sigma}$, pour $p^2=-\mu^2$, ne sont autres que les éléments de matrice des dimensions anormales des vertex coupés du quark et du gluon.

$$\begin{pmatrix} \mathcal{D}_{qq}^{\sigma} & \mathcal{D}_{qG}^{\sigma} \\ \mathcal{D}_{Gq}^{\sigma} & \mathcal{D}_{GG}^{\sigma} \end{pmatrix}_{p^2=-\mu^2} = \begin{pmatrix} \gamma_{qq}^{\sigma} & \gamma_{qG}^{\sigma} \\ \gamma_{Gq}^{\sigma} & \gamma_{GG}^{\sigma} \end{pmatrix} \equiv \overleftrightarrow{\gamma}^{\sigma} \tag{8}$$

L'équation (5), qui découle de la renormalisation, a une conséquence importante pour les noyaux $F_{\gamma q}$ et $F_{\gamma G}$.

$$\int \frac{d^4K}{K^4} \frac{1}{(2\pi)^4}\, \theta\left(\frac{\Delta K}{\Delta P}\right) \left(\frac{\Delta K}{\Delta P}\right)^{\sigma} \left[P^2 \frac{\partial}{\partial P^2} \vec{F} - \vec{F}\, \overleftrightarrow{\gamma}^{\sigma} \right]_{p^2=-\mu^2} = \vec{\mathcal{D}}^{\sigma} = \text{fini} \tag{9}$$

$\overleftrightarrow{\gamma}^{\sigma}$ est la matrice des dimensions anormales

$$\vec{F} = (F_{\gamma q}, F_{\gamma G}) \quad \text{et} \quad \vec{\mathcal{D}}^{\sigma} = (\mathcal{D}^{\sigma}_{\gamma q}, \mathcal{D}^{\sigma}_{\gamma G})_{p^2=-\mu^2} \tag{10}$$

Comme le membre de droite de l'équation (9) est fini, et que les variables $\rho = \frac{\Delta k}{\Delta p}$ et $z = \frac{k^2}{2pk}$ sont bornées par l'espace de phase $0 \leq \rho \leq z \leq 1$, nous trouvons pour un Q_0 suffisamment grand: (+)

$$\int_{Q_0^2}^{\infty} \frac{dk^2}{k^2} \int_0^1 d\rho\, \rho^{\sigma} \int_{\rho}^1 \frac{dz}{z}(1-\rho/z) \cdot \left[\frac{1}{z} p^2 \frac{\partial}{\partial p^2} \vec{F}_{as}\left(z, \frac{k^2}{\mu^2}, \frac{\rho^2}{\mu^2}\right) - \vec{F}_{as}\left(z, \frac{k^2}{\mu^2}, \frac{\rho^2}{\mu^2}\right) \cdot \overleftrightarrow{\gamma}^{\sigma} \right]_{p^2=-\mu^2} = \text{fini} \tag{11}$$

Cette équation (11) entraîne:

$$\left[p^2 \frac{\partial}{\partial p^2} \vec{F}^{\sigma}\left(\frac{q^2}{\mu^2}, \frac{p^2}{\mu^2}\right) - \vec{F}^{\sigma}\left(\frac{q^2}{\mu^2}, \frac{p^2}{\mu^2}\right) \overleftrightarrow{\gamma}^{\sigma} \right]_{p^2=-\mu^2} \xrightarrow{-q^2 \to \infty} \mathcal{O}\left(\frac{1}{q^2}\right) \tag{12a}$$

avec

$$\vec{F}^{\sigma}\left(\frac{q^2}{\mu^2}, \frac{p^2}{\mu^2}\right) = \int_0^1 dz\, z^{\sigma-1} \vec{F}\left(z, \frac{q^2}{\mu^2}, \frac{p^2}{\mu^2}\right) \tag{12b}.$$

En négligeant les corrections d'ordre $1/q^2$, et en utilisant le fait que $\vec{F}^{\sigma}$ est une quantité sans dimension, nous trouvons

$$p^2 \frac{\partial}{\partial p^2} \vec{F}^{\sigma}\left(\frac{q^2}{\mu^2}, \frac{p^2}{\mu^2}, \alpha\right) = \left(-\mu^2 \frac{\partial}{\partial \mu^2} - q^2 \frac{\partial}{\partial q^2}\right) \vec{F}^{\sigma}\left(\frac{q^2}{\mu^2}, \frac{p^2}{\mu^2}, \alpha\right) \tag{13a}$$

(+) La mesure d'intégration d^4k s'écrit, à l'aide des variables ρ, z et k^2, de la manière suivante: $d^4k = \frac{\pi}{2}(k^2)^2(1-\rho/z)\frac{1}{z^2}\, dz\, d\rho\, dk^2$.

Ou encore, en utilisant les équations (1a,b):

$$p^2 \frac{\partial}{\partial p^2} \vec{F}^{\sigma}\left(\frac{q^2}{\mu^2}, \frac{p^2}{\mu^2}, \alpha\right) = \left[\beta(\alpha)\frac{\partial}{\partial \alpha} - q^2 \frac{\partial}{\partial q^2}\right] \vec{F}^{\sigma}\left(\frac{q^2}{\mu^2}, \frac{p^2}{\mu^2}, \alpha\right), \tag{13b}$$

nous arrivons à l'équation suivante:

$$\left(D - \overleftrightarrow{\gamma}^{\sigma}\right)^{\perp} \vec{F}^{\sigma\perp}\left(\frac{q^2}{\mu^2}, 1, \alpha\right) = 0 \tag{14}$$

En combinant cette dernière avec l'équation (1a,b), nous observons que $\vec{F}^{\sigma}\left(\frac{q^2}{\mu^2}, \frac{p^2}{\mu^2}, \alpha\right)$ doit se factoriser, puisque l'équation (1a,b) admet comme solution générale:

$$\vec{F}^{\sigma}\left(\frac{q^2}{\mu^2}, \frac{p^2}{\mu^2}, \alpha\right) = \vec{F}^{\sigma}\left(\frac{q^2}{p^2}, 1, \alpha(p^2)\right) \tag{15a}$$

et l'équation (14)

$$\vec{F}^{\sigma}\left(\frac{q^2}{\mu^2}, 1, \alpha\right) = \vec{F}^{\sigma}\left(\frac{q^2}{p^2}, 1, \alpha(p^2)\right) . T\exp \int_{\alpha}^{\alpha(p^2)} \frac{d\alpha}{\beta(\alpha)} \overleftrightarrow{\gamma}(\alpha) \tag{15b}$$

Il vient:

$$\vec{F}^{\sigma}\left(\frac{q^2}{\mu^2}, \frac{p^2}{\mu^2}, \alpha\right) = \vec{F}^{\sigma}\left(\frac{q^2}{\mu^2}, 1, \alpha\right) . T\exp - \int_{\alpha}^{\alpha(p^2)} \frac{d\alpha}{\beta(\alpha)} \overleftrightarrow{\gamma}(\alpha) \tag{16}$$

Etant donné que les conditions de renormalisation choisies pour les vertex coupés d'un quark et un gluon sont celles fixées par l'équation (7), les quantités $\left(T\exp - \int_{\alpha}^{\alpha(p^2)} d\alpha/\beta(\alpha) \cdot \overleftrightarrow{\gamma}(\alpha)\right)_{ij}$ sont en fait les éléments de matrice des vertex coupés du quark et du gluon.

$$\Gamma_{ij}^{\sigma}\left(\frac{P^2}{\mu^2},\alpha\right) = \left(T\exp -\int_{\alpha}^{\alpha(P^2)} \frac{d\alpha}{\beta(\alpha)}\overleftrightarrow{\gamma}(\alpha)\right)_{ij} \quad , ij = q,G \qquad (17)$$

Nous avons donc démontré que les moments de Mellin des fonctions de structure, $F_{\gamma q}^{\sigma}$ et $F_{\gamma G}^{\sigma}$, sont des sommes de termes factorisés.

$$F_{\gamma q}^{\sigma}\left(\frac{q^2}{\mu^2},\frac{P^2}{\mu^2},\alpha\right) = C_{\gamma q}^{\sigma}\left(\frac{q^2}{\mu^2},\alpha\right).\Gamma_{qq}^{\sigma}\left(\frac{P^2}{\mu^2},\alpha\right) + C_{\gamma G}^{\sigma}\left(\frac{q^2}{\mu^2},\alpha\right)\Gamma_{Gq}^{\sigma}\left(\frac{P^2}{\mu^2},\alpha\right) \qquad (18a)$$

$$F_{\gamma G}^{\sigma}\left(\frac{q^2}{\mu^2},\frac{P^2}{\mu^2},\alpha\right) = C_{\gamma q}^{\sigma}\left(\frac{q^2}{\mu^2},\alpha\right).\Gamma_{qG}^{\sigma}\left(\frac{P^2}{\mu^2},\alpha\right) + C_{\gamma G}^{\sigma}\left(\frac{q^2}{\mu^2},\alpha\right).\Gamma_{GG}^{\sigma}\left(\frac{P^2}{\mu^2},\alpha\right) \qquad (18b)$$

avec $$C_{\gamma i}^{\sigma}\left(\frac{q^2}{\mu^2},\alpha\right) = F_{\gamma i}^{\sigma}\left(\frac{q^2}{\mu^2},1,\alpha\right) \qquad (18c).$$

Dans le cas physique d'une diffusion entre un état hadronique et un photon virtuel, on a:

$$F_{\gamma h}^{\sigma}\left(\frac{q^2}{\mu^2},\frac{P^2}{\mu^2},\alpha\right) = C_{\gamma q}^{\sigma}\left(\frac{q^2}{\mu^2},\alpha\right)\Gamma_{qh}^{\sigma}\left(\frac{P^2}{\mu^2},\alpha\right) + C_{\gamma G}^{\sigma}\left(\frac{q^2}{\mu^2},\alpha\right)\Gamma_{Gh}^{\sigma}\left(\frac{P^2}{\mu^2},\alpha\right) \qquad (19)$$

Les vertex coupés, $\Gamma_{qh}^{\sigma}(P^2/\mu^2,\alpha)$ et $\Gamma_{Gh}^{\sigma}(P^2/\mu^2,\alpha)$, représentent les transformées de Mellin des distributions de quarks et de gluons dans l'état hadronique, pour $Q = \mu$

$$\langle q^h(\mu)\rangle_\sigma = \Gamma_{qh}^\sigma\left(P^2/\mu^2, \alpha_r\right) \tag{20a}$$

$$\langle G^h(\mu)\rangle_\sigma = \Gamma_{Gh}^\sigma\left(P^2/\mu^2, \alpha_r\right) \tag{20b}$$

Puisque $F_{\gamma h}^\sigma$ satisfait à l'équation:

$$D F_{\gamma h}^\sigma\left(\frac{q^2}{\mu^2}, \frac{P^2}{\mu^2}, \alpha_r\right) = 0 \tag{21}$$

on peut remplacer μ^2 par $-q^2 = Q^2$, en remplaçant en même temps α_r par $\alpha(Q)$, l'équation (19) devient alors:

$$F_{\gamma h}^\sigma(P,q) = C_{\gamma q}^\sigma\left(-1, \alpha(Q)\right)\Gamma_{qh}^\sigma\left(-\frac{P^2}{Q^2}, \alpha(Q)\right) + C_{\gamma G}^\sigma\left(-1, \alpha(Q)\right)\Gamma_{Gh}^\sigma\left(-\frac{P^2}{Q^2}, \alpha(Q)\right) \tag{22}$$

Ou encore, en faisant la transformation de Mellin inverse,(+)

$$F_{\gamma h}(x, P, q) = E_{\gamma q}(x, \alpha(Q)) \otimes q^h(x, Q) + E_{\gamma G}(x, \alpha(Q)) \otimes G^h(x, Q) \tag{23}$$

qui représente l'équation(III.3)du modèle des partons généralisé.

Comme $\Gamma_{ih}^\sigma\left(\frac{P^2}{\mu^2}, \alpha_r\right)$ satisfait à l'équation:

$$D\,\Gamma_{ih}^\sigma\left(P^2/\mu^2, \alpha_r\right) = \sum_j \gamma_{ij}^\sigma(\alpha_r)\,\Gamma_{jh}^\sigma\left(\frac{P^2}{\mu^2}, \alpha_r\right) \tag{24}$$

(+) - Le produit de convolution $\otimes$ est défini comme suit:

$$A(x) \otimes B(x) = \int_x^1 \frac{dy}{y} A\left(\frac{x}{y}\right) B(y)$$

- $\langle E_{\gamma i}(\alpha(Q))\rangle_\sigma = C_{\gamma q}^\sigma\left(-1, \alpha(Q)\right)$

et $\langle P_{ij}(\alpha(Q))\rangle_\sigma = -2\,\gamma_{ij}^\sigma(\alpha(Q))$

on obtient une équation simple pour les distributions de quarks et de gluons:

$$Q\frac{\partial}{\partial Q} q_i(x,Q) = \sum_j P_{ij}(x,\alpha(Q)) \otimes q_j(x,Q) \quad , \; i,j = q,\bar{q},G \qquad (25)$$

Cette équation donne aux noyaux $P_{ij}(x,\alpha(Q))$ une interprétation probabiliste simple ; $P_{ij}(x)$ est la probabilité de trouver un parton i dans un parton j, avec une fraction de l'impulsion longitudinale X, lorsqu'on fait varier l'échelle $\log Q$ d'une unité.

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- V -

CORRECTIONS D'ORDRES SUPERIEURS

DANS UNE THEORIE SCALAIRE

En prenant l'exemple de la théorie scalaire $[\phi^3]_6$, nous allons développer une technique de calcul simple, pour les vertex coupés et les fonctions coefficients. Nous démontrerons l'équivalence entre le formalisme des vertex coupés et le développement sur le cône de lumière dans le cas de la diffusion et nous vérifierons cette équivalence au niveau des diagrammes à deux boucles.

La régularisation dimensionnelle permet d'expliciter la dépendance dans les conditions de renormalisation choisies pour les vertex coupés; de par le choix des variables d'espace de phase, les calculs deviennent clairs, et les relation entre quantités de genre espace et de genre temps évidentes.

Nous donnerons des exemples explicites et présenterons les résultats du calcul à l'ordre g^4 pour les vertex coupés (du genre temps et espace).

Enfin, nous discuterons les relations entre les processus de diffusion et d'annihilation. Nous démontrerons aussi l'équivalence entre le formalisme des vertex coupés et le développement des opérateurs de "twist" minimum.

Le choix de $[\phi^3]_6$, à ce stade, nous permet d'étudier les ordres supérieurs tout en évitant l'algèbre très difficile de la chromodynamique quantique. L'application de ce formalisme à la chromodynamique quantique sera l'objet des chapitres suivants.

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PARTON MODEL INTERPRETATION OF THE CUT VERTEX FORMALISM

par

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ABSTRACT

We propose a parton model interpretation of the cut vertex formalism recently developed by A.H. Mueller. We show that this interpretation extends to all orders in the running coupling constant $\bar{\alpha}(t)$, the Altarelli-Parisi picture in both the time-like and space-like region. We show the equivalence with the light cone expansion method in the space-like region and we verify this equivalence by explicit calculations in $[\phi^3]_6$ up to the two-loop level both for the space-like cut vertices and the matrix elements of minimal twist Wilson operators.

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1. INTRODUCTION

Important progress has been made recently in understanding the short distance hadronic structure in the framework of quantum chromodynamics [1].

We have now a better understanding of why the parton model ideas are so succesful in describing hard scattering processes. The parton model ideas are based on the assumption that in hard scattering processes two classes of scales are involved, one of fairly small scales which are connected with the probing currents and another one of large scales which are connected with the formation of hadrons.

The important development is that we have now methods to appropriately separate the dynamics of the two regions, the short distance dynamics being calculable [2,3,4]. Recent diagrammatic studies of the soft and collinear divergences which appear in the perturbative treatment of the cross-sections for hard scattering processes in gauge theories [2,3], reveal that moments of these cross-sections indeed factor into a product of universal factors (i.e. independent of the process) for every initial or final parton, which participates in the process, factors which contain the large distance dynamics and into a kernel which is calculable in perturbation expansion in the coupling constant and which depends only on the short distance dynamics of the specific process.

In ref. [4] an extension of the light cone expansion has been developed and it has been shown that the calculable hard scattering kernel satisfies a renormalization group equation (RGE) which greatly facilitates the summation of the U.V. logarithms of all orders of the perturbative expansion in the strong coupling constant.

In this work we propose an interpretation of the cut vertex formalism of ref.[4] for the space-like and time-like region, in terms of parton densities and fragmentation functions, which extends to all orders in $\bar{\alpha}(t)$ the Altarelli-Parisi (A-P) picture [5a], for both the space-like and the time-like regions.

In sec.2 we review the cut vertex formalism.

In sec.3-4 we define the parton densities and we discuss the generalized A-P equation.

In sec.5 we define the parton fragmentation functions and the related A-P equation. In appendix A we give the results for space-like cut vertices up to two loops in $[\phi^3]_6$ as well as the one loop coefficient functions for the current $J(x) = :\phi^2(x):$. This calculation checks the identity of space-like cut vertices and the analytic continuation of matrix elements of minimal twist Wilson operators in the spin variable. Apart from this check we present a heuristic argument about the validity of this identity to all orders in the coupling constant.

In appendix B we give the renormalization condition appropriate for the validity of the Gribov-Lipatov reciprocity relation [14] for the anomalous dimensions of the space-like and time-like cut vertices to all orders in the coupling constant.

Our formalism is easily extended in other field theories (for example gauge field theories) with more than one kind of partons.

2 . THE SPACE LIKE CUT VERTICES

In this section we review the cut vertex formalism [4], in the framework of the model field theory $[\phi^3]_6$. This theory shares with QCD the property of being asymptotically free [6], but which is of a much milder infrared and mass singularity structure. The bare Lagrangian for a massless ϕ field is :

$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi_0)^2 + g_0/3!\ \phi_0^3. \tag{2.1}$$

We shall discuss the forward Compton scattering of a current $J(q)$, with a large space-like momentum q, on a particle $\tilde{\phi}(p)$, whose momentum is slightly off mass-shell and space-like (fig.1) :

$$J(q) + \tilde{\phi}(p) \to J(q) + \tilde{\phi}(p).$$

The total cross section for the process,

$$J(q) + \tilde{\phi}(p) \to \text{anything},$$

is related to the absorptive part of the Compton scattering amplitude(*):

$$Q^2 .\ W(x,Q^2,p^2) \equiv \frac{1}{\pi} \mathrm{Im} \int d^6y\ e^{iq.y} \langle 0|T(\tilde{\phi}(-p)J(y)J(0)\tilde{\phi}(p))|0\rangle . \Delta_F(p^2)^{-1}, \tag{2.2}$$

(*) $\Delta_F(p^2) = \langle 0|T(\tilde{\phi}(-p)\ \tilde{\phi}(p))|0\rangle$.

where,

$$x = -q^2/2p.q \quad , \quad 0 < x < 1,$$

and we choose the reference frame where q has only $q_3 \neq 0$, $q = (0, -Q, \vec{0})$ and $p = (p_0, p_3, \vec{0})$ with $p_- \equiv p_0 \pm p_3 \gtrless 0$.

In ref. 4 it is shown that in the Bjorken limit, x fixed and $Q^2 \to +\infty$, the moments of the structure function $W(x, Q^2, p^2)$ factorize in the following way, ($Re\sigma > 1$),

$$\int_0^1 dx\, x^{\sigma-1}\, W(x,Q^2,p^2) = A_\sigma(p^2/\mu^2,\alpha)\, C_\sigma(Q^2/\mu^2,\alpha) \ , \tag{2.3}$$

where,

$$C_\sigma(Q^2/\mu^2,\alpha) \equiv \int_0^1 dx\, x^{\sigma-1}\, W(x,Q^2,-\mu^2) \tag{2.4}$$

and

$$A_\sigma(p^2/\mu^2,\alpha) \equiv p_+^{-\sigma+1} \int \frac{d^6k}{(2\pi)^6} \frac{k_+^{\sigma-1}\, \theta(k_+)}{(k^2)^2}\, T(p,k) \ . \tag{2.5}$$

Here μ is the renormalization point, $g(\mu)$ the renormalized coupling constant and $\alpha = g^2(\mu) / (4\pi)^3$. The function $T(p,k)$ is the absorptive part of the renormalized 4-point function for the process (fig.2),

$$\tilde{\phi}(-k) + \tilde{\phi}(p) \to \tilde{\phi}(p) + \tilde{\phi}(-k),$$

and the k-integration in rel.(2.5) is appropriately renormalized.

The expression

$$\int \frac{d^6k}{(2\pi)^6} \frac{k_+^{\sigma-1}\, \theta(k_+)}{(k^2)^2}\, T(p,k) = p_+^{\sigma-1}\, A_\sigma(p^2/\mu^2,\alpha), \tag{2.6}$$

is called space-like cut vertex and it may be described as the radiative correction to the bare cut vertex $p_+^{\sigma-1}\, \theta(k_+)$ (fig. 3).

In ref. 4 the U.V. divergences of the cut vertices are substracted at the point $\hat{p}$:

$$\hat{p}_+ = 0 \quad , \quad \hat{p}_- = p_- \quad , \quad \hat{p}_\perp^2 = 0$$

We work with a massless field $\phi < 0$ we have to substract at a different point with $p^2 \neq 0$. We choose the point $\hat{p}$:

$$\hat{p}_+ = p_+ \quad , \quad \hat{p}_- = 0 \quad , \quad \hat{p}_\perp^2 = \mu^2 \ .$$

These divergences are shown to give a multiplicative renormalization to the bare cut vertex [4]. This means that the renormalized cut vertices satisfy the renormalization group equation(*) :

$$(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha)\ \alpha \frac{\alpha}{\partial \alpha} + \gamma_\sigma(\alpha)) - A_\sigma(p^2/\mu^2,\alpha) = 0 \ , \tag{2.7}$$

where γ_ϕ is the anomalous dimension of the ϕ field.

The factorization implies that the coefficient function $C_\sigma(Q^2/\mu^2,\alpha)$ satisfies the differential equation :

$$(\mu^2 \frac{\partial}{\partial \mu^2} + \beta(\alpha)\ \alpha \frac{\alpha}{\partial \alpha} - \gamma_\sigma(\alpha))\ C_\sigma(Q^2/\mu^2,\alpha) = 0 \ , \tag{2.8}$$

whose solution is :

$$C_\sigma(Q^2/\mu^2,\alpha) = C_\sigma(1,\bar{\alpha})\ \exp -\int_\alpha^{\bar{\alpha}} \frac{d\alpha'}{\alpha'} \frac{\gamma_\sigma(\alpha')}{\beta(\alpha')} \ , \tag{2.9}$$

with :

$$Q^2 \frac{d}{dQ^2}\ \bar{\alpha}(Q^2) = \bar{\alpha}(Q^2)\ \beta(\bar{\alpha}(Q^2)) \ , \ \bar{\alpha}(Q^2 = \mu^2) = \alpha \ .$$

The coefficient function $C_\sigma(1,\bar{\alpha})$ as well as $\gamma_\sigma(\bar{\alpha})$ can be computed in perturbation expansion in $\bar{\alpha}$. If we known $\beta(\bar{\alpha})$ and $\gamma_\sigma(\bar{\alpha})$ up to order $\bar{\alpha}^m$ and $C_\sigma(1,\bar{\alpha})$ up to order $\bar{\alpha}^{m-1}$ we can sum up the m-first leading logarithms in Q^2 of all orders in α for the function $C_\sigma(Q^2/\mu^2,\alpha)$.

3. SPACE LIKE CUT VERTICES AS PARTON DENSITIES

In this section we propose a diagrammatic definition of parton densities which make possible a probabilistic description of the hard scattering pro-

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(*)This equation is correct if the current J(x) is conserved, otherwise $\gamma_\sigma \to \gamma_\sigma + \gamma_{JJ}$, where γ_{JJ} is the anomalous dimensions of the operator J(x)J(0).

cesses and which extends to all orders in $\bar{\alpha}(t)$ the A-P picture [5a]. For the leading logarithmic approximation this has been done rencently in gauge theories [7,8].

By parton density of a parton we mean the probability to find inside a parton $\tilde{\phi}(p)$, at scale Q^{-1}, another parton with longitudinal momentum along $\vec{p}$ equal to $x.\vec{p}$.

It is suggestive that the diagrams contributing to the parton density are those of the type of fig.2, where an integration over the virtual momentum k is implied with the following constraints : We include a delta function $\delta(x - k_+/p_+)$, which fixes the longitudinal momentum and a $\theta(k_+)$ functions which ensures that $x > 0$. We include also the propagators of the external legs of momentum k, $(1/k^2)^2$, as if the parton of momentum k is probed a current [9](fig.4).

We note that the phase space constraints associated with the cuts of the diagrams of fig.2 ensure that $x \leq 1$.

Let $T(p^2,k^2,p.k,\mu^2,\alpha)$ denote the absorptive part of the renormalized forward four point function(*) for the process,

$$\tilde{\phi}(p) + \tilde{\phi}(-k) \rightarrow \tilde{\phi}(p) + \tilde{\phi}(-k),$$

which corresponds to the sum over all possible cuts which do not separate any external leg of the contributing graphs.

The above mentioned integration over the virtual momentum k introduces UV divergences, so we have to renormalize the resulting expression choosing a regularization for this integration and a renormalization condition at a point $p^2 = -\mu'^2 < 0$.

We choose the point $\mu' = \mu$, where μ is the renormalization point we use for the two and three point functions of the $[\phi^3]_6$ theory.

The renormalized integral evaluated at $\mu = Q$ and $\alpha = \bar{\alpha}(Q^2)$ we define to be the parton density of the parton of momentum p, probed by the current of momentum Q^2 :

(*) Amputated with the free (square root of the full) propagator for the external legs of momentum k(p).

$$q(x, \frac{p^2}{Q^2}, \bar{\alpha}(Q^2)) \equiv \left\{ \int \frac{d^6k}{(2\pi)^6} \frac{\theta(k_+)\delta(x-k_+/p_+)}{(k^2)^2} T(p^2,k^2,p.k,\mu^2,\alpha) \right\}\Big|_{\substack{\text{renorm.} \\ \alpha t p^2=-\mu'^2}} \Big|_{\substack{\mu=\mu'=Q \\ \alpha=\bar{\alpha}(Q^2)}} . \quad (3.1)$$

We show now how this definition really extends the A-P picture to all orders in $\bar{\alpha}$. Observed first that the moments of the parton density (3.1) are exactly the space-like cut vertices (rel(2.5)), (Reσ > 1),

$$\int_0^1 dx\, x^{\sigma-1}\, q(x,p^2/Q^2,\bar{\alpha}(Q^2)) = A_\sigma(p^2/Q^2,\bar{\alpha}(Q^2)) \equiv q_\sigma(Q^2). \quad (3.2)$$

Second since $A_\sigma(p^2/\mu^2,\alpha)$ satisfies the RGE (2.7), performing the dilatation,

$$p^2/\mu^2 \to p^2/\mu^2 \,.\, \mu^2/Q^2 = p^2/Q^2, \quad (3.3)$$

we find :

$$A_\sigma(p^2/Q^2,\bar{\alpha}(Q^2)) = A_\sigma(p^2/\mu^2,\alpha)\, \exp -\int_\alpha^{\bar{\alpha}'} \frac{d\alpha'}{\alpha'} \frac{\gamma_\sigma(\alpha')}{\beta(\alpha')}, \quad (3.4)$$

which shows that the parton density (3.1) is renormalization group invariant :

$$[\mu^2 \frac{\partial}{\partial\mu^2} + \beta(\alpha)\, \alpha \frac{\partial}{\partial\alpha}]\, q(x,p^2/Q^2)\, \bar{\alpha}(Q^2)) = 0 .$$

We take now the derivative, $Q^2 \frac{\partial}{\partial Q^2}$, both sides in eq. (3.4),

$$Q^2 \frac{\partial}{\partial Q^2} q_\sigma(Q^2) = -\gamma_\sigma(\bar{\alpha}(Q^2))\, q_\sigma(Q^2), \quad (3.5)$$

or

$$Q^2 \frac{\partial}{\partial Q^2}\, q(x,p^2/Q^2,\, \bar{\alpha}(Q^2)) = \int_x^1 \frac{dy}{y}\, P(\frac{x}{y},\bar{\alpha}(Q^2))\, q(y,P^2/Q^2,\bar{\alpha}(Q^2)), \quad (3.6)$$

where $P(x,\bar{\alpha})$ is defined as :

$$\int_0^1 dx\, x^{\sigma-1}\, P(x,\bar{\alpha}) \equiv -\gamma_\sigma(\bar{\alpha}). \quad (3.7)$$

Equation (3.6) extends to all orders in $\bar{\alpha}$ the A-P equation and it has the same probabilistic interpretation since $q(x,P^2/Q^2,\bar{\alpha}(Q^2))$ is positive by contruction and $P(x,\bar{\alpha})$ represents its relative variation per unit dx and unit d log Q^2/μ^2 (*). This equation has been derived also in the light cone expansion formalism in ref. 5b-5c, but the advantage of the derivation given here is that we start directly with a probabilistic definition of the parton densities.

The same formalism will be used to define the fragmentation functions (see.5).

In the following section we discuss the physical applications of eq.3.6.

4 . HADRONIC PARTON DENSITIES

The structure function of hadron are measured in experiments of deep-inelastic lepton-hadron scattering and the comparison with the theory is usually done at the level of moments [10].

Let $F_h(x,Q^2)$ be a non-singled (NS) structure functions, $(\frac{F_2^{ep}-F_2^{en}}{x}, F_3^{vp})$, then its moments are given by the relation [4]:

$$\int_0^1 dx\, x^{\sigma-1}\, F_h(x,Q^2) = A_\sigma^h . C_\sigma^{NS}(1,\bar{\alpha})\, \exp -\int_\alpha^{\bar{\alpha}} \frac{d\alpha'}{\alpha'} \frac{\gamma_\sigma^{NS}(\alpha')}{\beta(\alpha')}\,, \tag{4.1}$$

where $C_\sigma^{NS}(1,\alpha)$, $\gamma_\sigma^{NS}(\alpha)$ are calculable in perturbation in α and A_σ^h are uncalculable but measurable constants. We define an effective charge $e(x,\bar{\alpha})$ and the (NS) parton density of the hadron h as follows :

$$\int_0^1 dx\, x^{\sigma-1}\, e(x,\bar{\alpha}) \equiv C_\sigma^{NS}(1,\bar{\alpha})\,, \tag{4.2}$$

$$\int_0^1 dx\, x^{\sigma-1}\, q_h(x,\bar{\alpha}) \equiv A_\sigma^h \cdot \exp -\int_\alpha^{\bar{\alpha}} \frac{d\alpha'}{\alpha'} \frac{\gamma_\sigma^{NS}(\alpha')}{\beta(\alpha')}\,. \tag{4.3}$$

Equation (4.1) can be inverted to give :

$$F(x,Q^2) = \int_x^1 \frac{dy}{y}\, e(\frac{x}{y},\alpha)\; q_h(y,\bar{\alpha}) \equiv e(x,\bar{\alpha}) \otimes q_h(x,\bar{\alpha}). \tag{4.4}$$

(*) $P(x,\bar{\alpha})$ has to satisfy certain sum rules which will be discussed in sec.5 [5a,5b].

The RG invariant hadronic parton density (4.3) has a probabilistic interpretation analogous to the one discussed in see.3 for partons : it satisfies the same generalized A-P evolution equation :

$$Q^2 \frac{\partial}{\partial Q^2} q_h(x,\bar{\alpha}) = P(x,\bar{\alpha}) \otimes q_h(x,\bar{\alpha}), \tag{4.5}$$

with $P(x,\bar{\alpha})$ defined in (3.5).

If we combine equations 4.4 and 4.5 we find the evolution equation for the structure function,(*)

$$Q^2 \frac{\partial}{\partial Q^2} F_h(x,Q^2) = \left[e^{-1}(x,\bar{\alpha}) \otimes Q^2 \frac{\partial}{\partial Q^2} e(x,\bar{\alpha}) + P(x,\bar{\alpha}) \right] \otimes F_h(x,Q^2). \tag{4.6}$$

Given the structure function $F_h(x,Q^2)$ and its derivative $Q^2 \frac{\partial}{\partial Q^2} F_h(x,Q^2)$ at every $x > x_0$ and Q^2 one may compare the two sides of eq.(4.6), or given $F_h(x,Q_0^2)$ for $x > x_0$ one may solve the evolution equation.(4.6) and find $F(x,Q^2)$ for every $Q^2 > Q_0^2$ and $x > x_0$. Notice the important point that the convolution on the right hand side of eq. 4.7 needs data only for $x \geq x_0$ in order to find the Q^2 dependence of $F(Q^2,x)$ for $x \geq x_0$. This is definitely an advantage over the moment equations where we need to know the structure function down to $x = 0$.

The hadronic parton density,

$$q_h = e^{-1} \otimes F_h, \tag{4.7}$$

satisfies naive parton model sum rules. Indeed if a specific anomalous dimension vanishes then eq.(4.5) implies that the corresponding moment of q_h does not dependent on Q^2 and we shall discuss in see.5 why this constant value is fixed to the naive parton model value.

In ref.[12] the hadronic parton densities are defined to be the structure functions themselves (for example $\bar{q}+q = F_2^{vp}$). Under this definition the renormalization prescription dependence of the relevant coefficient functions and parton densities is absent.

(*) We define e^{-1} as : $\int_x^1 \frac{dy}{y} e(\frac{x}{y},\bar{\alpha})\ e^{-1}(y,\bar{\alpha}) = \delta(1-x)$.

On the other hand there is an explicit dependence of the so defined parton densities on the current, simple evolution equations and naive parton model sum rules (except the Adler sum rule) do not exist beyond the leading logarithmic approximation.

We are aware of the fact that according to our definition the parton densities are renormalization scheme dependent (although inside our renormalization scheme they are renormalization group invariant) and in order to test the universality hypothesis we have to use the same renormalization scheme for all hard scattering processes. On the other hand easily we can construct renormalization scheme (RS) independent quantities [13]. We give here two examples. If F, G are two different non-singlet structure functions and e_F, e_G are their corresponding coefficient functions the following two expressions are RS independent quantities :

$$e_F(x,\bar{\alpha}) \otimes e_G^{-1}(x,\bar{\alpha}) = F(x,Q^2) \otimes G^{-1}(x,Q^2) \ , \tag{4.8}$$

$$(Q^2 \frac{\partial}{\partial Q^2} e_F(x,\bar{\alpha})) \otimes e_F^{-1}(x,\bar{\alpha}) + P(x,\bar{\alpha}) = (Q^2 \frac{\partial}{\partial Q^2} F(x,Q^2)) \otimes F^{-1}(x,Q^2) . \tag{4.9}$$

For an application of a similar formalism (based on the light cone expansion) in the extraction of the gluon density of the proton as well as in the study of the scaling violation for the non-singlet structure function F_2^{ep-en}, we refer to ref.[11].

5. TIME LIKE CUT VERTICES AS FRAGMENTATION FUNCTIONS

In this section we define the fragmentation function of a parton to a parton as well as the fragmentation function of a parton to a hadron [17,18]. These definitions are in close analogy with the space-like case and therefore we shall be less explicit than in the previous sections.

Let $\overline{W}(x,Q^2,p^2)$ be the fragmentation function (or annihilation structure function) of a current $J(q)$, $q^2 > 0$, decaying into a field quantum $\tilde{\phi}(p)$, $p^2 \gtrless 0$,

$$Q^2\overline{W}(x,Q^2,p^2) \equiv \frac{1}{\pi} \int d^6y \, e^{iqy} < 0|J^+(y) \, \tilde{\phi}(-p) \, \tilde{\phi}(p) \, J(0)|0 > |\Delta_F(p^2)|^{-1}, \tag{5.1}$$

where $x = Q^2/2p.q$, $1 < x < \infty$ and we choose the frame where,

$q = (Q, 0, \vec{0})$

$p = (p_0, p_3, \vec{0})$

and $p_{\pm} < 0$.

In ref.4 it is shown that the Q^2 and p^2 dependences of the moments of $\bar{W}(x,Q^2,p^2)$ factorize :

$$\int_1^\infty dx\ x^{-\sigma-1}\ \bar{W}(x,Q^2,p^2) = \bar{A}_\sigma(p^2/\mu^2,\alpha)\ \bar{C}_\sigma(Q^2/\mu^2,\alpha), \tag{5.2}$$

where

$$\bar{C}_\sigma(Q^2/\mu^2,\alpha) = \int_1^\infty dx\ x^{-\sigma-1}\ \bar{W}(x,Q^2,\mu^2) \tag{5.3}$$

and apart from renormalization problems $\bar{A}_\sigma$ are defined by :

$$\bar{A}_\sigma(p^2/\mu^2,\alpha) = p_+^{\sigma+1} \int \frac{d^6k}{(2\pi)^6}\ \frac{k_+^{-\sigma-1}}{(k^2)^2}\ \bar{T}(p,k). \tag{5.4}$$

The cut four-point function $\bar{T}(p,k)$, $(p^2 > 0)$, (fig. 2b) is defined as in the space-like case (sec.3). The renormalization of all above defined quantities has to be done as in the space-like case but the time-like cut vertices are substracted at the time-like point $p^2 = \mu'^2$. They are multiplicatively renormalized and satisfy the RGE [4]:

$$\left(\mu^2 \frac{\partial}{\partial\mu^2} + \beta(\alpha)\ \alpha \frac{\alpha}{\partial\alpha} + \bar{\gamma}_\sigma(\alpha)\right) \bar{A}\ (p^2/\mu^2,\alpha) = 0\ . \tag{5.5}$$

This implies the following differential equation for the coefficient functions $\bar{C}_\sigma(Q^2/\mu^2,\alpha)$:

$$\left(\mu^2 \frac{\partial}{\partial\mu^2} + \beta(\alpha)\ \alpha \frac{\partial}{\partial\alpha} - \bar{\gamma}_\sigma(\alpha)\right) \bar{C}_\sigma(Q^2/\mu^2,\alpha) = 0\ . \tag{5.6}$$

We define the fragmentation function of parton $\tilde{\phi}(p)$ to a parton $\tilde{\phi}(k)$ $(p^2,k^2 > 0)$ in analogy with the parton density case rel.(3.1) :

$$D(x,p^2/Q^2,\bar{\alpha}(Q^2)) \overset{\times}{=} \left\{ \int \frac{d^6k}{(2\pi)^6}\ \frac{\delta(x-p_+/k_+)}{(k^2)^2}\ \bar{T}(p^2,k^2,p.k,\mu^2,\alpha) \right\}_{\substack{\text{renorm.} \\ \alpha t\ p^2=\mu'^2}}^{\substack{\mu=\underline{\mu}'=Q \\ \alpha=\alpha}}\ . \tag{5.7}$$

The moments of D are the time-like cut vertices for $\mu = Q$ and $\alpha = \bar{\alpha}$, therefore D satisfies an A-P type probabilistic equation :

$$Q^2 \frac{\partial}{\partial Q^2} D(x,P^2/Q^2,\bar{\alpha}(Q^2)) = \int_1^x \frac{dy}{y} \bar{P}(\frac{x}{y},\bar{\alpha}(Q^2))\ D(y,P^2/Q^2,\bar{\alpha}(Q^2)), \qquad (5.8)$$

where $\bar{P}(x,\bar{\alpha})$ is defined as :

$$\int_1^\infty dx\ x^{-\sigma-1}\ \bar{P}(x,\bar{\alpha}) \equiv -\bar{\gamma}_\sigma(\alpha). \qquad (5.9)$$

Similarly the non-singlet fragmentation function of a parton to a hadron $D_h(x,\bar{\alpha})$ is defined as :

$$\int_1^\infty dx\ x^{-\sigma-1}\ D_h(x,\bar{\alpha}) \equiv \bar{A}_\sigma^h \exp -\int_\alpha^{\bar{\alpha}} \frac{d\alpha'}{\alpha'} \frac{\bar{\gamma}_\sigma(\alpha')}{\beta(\alpha')}, \qquad (5.10)$$

where $\bar{A}_\sigma^h$ are uncalculable but measurable constants defined through the moment equation for a non-singlet annihilation structure function, $\bar{W}_{NS}$ (for example $(W_2^{e^+e^- \to \bar{p}x} - W_2^{e^+e^- \to \bar{n}x})/x$) :

$$\int_1^\infty dx\ x^{-\sigma-1}\ \bar{W}_{NS}(x,Q^2) = \bar{A}_\sigma^h\ \bar{C}_\sigma^{NS}(1,\bar{\alpha}) \exp -\int_\alpha^{\bar{\alpha}} \frac{d\alpha'}{\alpha'} \frac{\bar{\gamma}_\alpha(\alpha')}{\beta(\alpha')}. \qquad (5.11)$$

From relation 5.9 and 5.10 we obtain the evolution equation for $D_h(x,\bar{\alpha})$, true to all orders in $\bar{\alpha}$:

$$Q^2 \frac{\partial}{\partial Q^2}\ D_h(x,\bar{\alpha}) = \int_1^x \frac{dy}{y} \bar{P}(\frac{x}{y},\bar{\alpha})\ D_h(y,Q^2). \qquad (5.12)$$

Equation 5.12 has been studied first in reference [17] in the leading logarithmic approximation, i.e. first order in $\bar{\alpha}$ for $\bar{P}(x,\bar{\alpha})$.

In order to be consistent with a parton model interpretation we require that the fragmentation function (5.7) satisfies sum rules. To realize this to all orders in $\bar{\alpha}$ we choose to relate the transition probability densities for the space-like case $P(x,\bar{\alpha})$ rel.(3.7) and the time-like case $\bar{P}(x,\bar{\alpha})$ rel.(5.9) by the Gribov-Lipatov (G-L) reciprocity relation(*) [14] :

(*) b = 3 for $[\phi^3]_6$.

$$\bar{P}(x,\bar{\alpha}) = \frac{1}{x^b} P(\frac{1}{x},\bar{\alpha}) \ , \tag{5.13}$$

which is known to be true in the one loop approximation. This relation can be imposed on $\bar{P}(x,\bar{\alpha})$ or $\bar{\gamma}_\sigma(\bar{\alpha})$ to all orders in $\bar{\alpha}$ because the coefficients $\bar{\gamma}_\sigma^{(i)}$ of the expansion of $\bar{\gamma}_\sigma$ in $\bar{\alpha}$,

$$\bar{\gamma}_\sigma = \gamma_\sigma^{(1)}\bar{\alpha} + \gamma_\sigma^{(2)}\bar{\alpha}^2 + \ldots,$$

are renormalization prescription dependent for $i \neq 1$. In appendix B we give the appropriate renormalization condition for the time-like cut vertices which ensure the validity of rel. 5.13 to all orders in $\bar{\alpha}$. At this point we must emphasize that the relation (5.13) does not imply that the G-L relation holds for the structure functions $W(x,Q^2)$, $\bar{W}(x,Q^2)$ beyond the leading log approximation. In fact we have shown in ref.[15] that this is not true in $[\phi^3]_6$ at the two-loop or next to leading log approximation. We now come to the implications of relation 5.13 for the parton densities $q_h(x,\bar{\alpha})$ (4.3) and fragmentation functions $D_h(x,\bar{\alpha})$ (5.10).

This relation transfers the sum rules of the space like region for $q_h(x,\bar{\alpha})$ to sum rules for $D_h(x,\bar{\alpha})$. Indeed for each vanishing moment of $P(x,\bar{\alpha})$ we have a vanishing moment of $\bar{P}(x,\bar{\alpha})$. However we do not know how to calculate the numerical values of the Q^2 independent moments of $D_h(x,\bar{\alpha})$, which are not matrix elements of local operators [4]. On the contrary, as we show in the appendix A, the space-like cut vertices are the analytic continuation in the spin variable of matrix elements of Wilson operators. So the Q^2 independent integer moments of $q_h(x,\bar{\alpha})$ are the matrix elements of conserved local operators with well defined eigenvalues on hadronic states (for example the energy momentum tensor or the baryonic charge).

We close this section by making the following remark . If for some Q_0^2 the Gribov-Lipatov reciprocity relation holds between D_h and q_h :

$$D_h(x,Q_0^2) = \frac{1}{x^b} q_h(\frac{1}{x},Q_0^2) \ , \tag{5.14}$$

then eq. 5.12 and 5.13 imply that it holds for every $Q^2 \neq Q_0^2$.

CONCLUSION

In this work we presented an alternative way of looking at the cut vertex formalism which we think shows the existence of a parton model interpretation of the factorization of large distance from the small distance dynamics in the hard scattering processes, $e^- h \to e + x$, $e^+e^- \to \bar{h} + x$. The cut vertices are the objects which admit a probabilistic interpretation and satisfy simple evolution equations and naive parton model sum rules. We checked up to two loops the identity of the cut vertex and light cone expansion methods in the space-like region.

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APPENDIX A

This appendix gives a heuristic demonstration that the space-like cut vertices are the analytic continuation in the spin variable of the matrix elements of minimal twist Wilson operators.

We check this statement by an explicit two loop calculation for both objects. The details of this calculation will appear in ref [16].

The minimal twist Wilson operators,

$$O_n^{\mu_1,\cdots\mu_n} = (-i)^n \phi(0)\, \partial^{\mu_1}\ldots\partial^{\mu_n} \phi(0), \tag{A.1}$$

can be analytically continued in the index n by considering the contraction of the indices $\mu_1,\ldots,\mu_n$ with the indices of a light-like vector Δ, $\Delta^2 = 0$,

$$O_n \equiv (-i)^n \phi(0)\, (\Delta.\partial)^n \phi(0) \rightarrow O_\sigma = (-i)^\sigma \phi(0)\, (\Delta.\partial)^\sigma \phi(0).$$

We choose $\Delta_0 = 1$, $\Delta_3 = -1$, $\Delta_\perp = 0$. The calculation of the two point function $<|T(\tilde{\phi}(-p)\ \tilde{O}_\sigma(0)\ \tilde{\phi}(p))|0>$ involves diagram of the type of fig.5 :

$$\Gamma_{O_{\sigma-1}} \equiv P_+^{-\sigma+1} \int \frac{d^6k}{(2\pi)^6}\, k_+^{\sigma-1}\, (G^{(2)}(k^2+i\varepsilon))^2 .\, G^{(4)}(p,k), \tag{A.2}$$

where $G^{(2)}$, $G^{(4)}$ are the two and four point functions, while for the space-like cut vertices we must calculate the diagram of fig.3 :

$$A_\sigma = P_+^{-\sigma+1} \int \frac{d^6k}{(2\pi)^6}\, k_+^{\sigma-1}\, \theta(k_+)\, [G^{(2)}(k^2+i\varepsilon)]^2\, \mathrm{Abs}_{(p-k)^2}\, G^{(4)}(k,p). \tag{A.3}$$

In rel.A.3, $\mathrm{Abs}_{(p-k)^2}\, G^{(4)}(k,p)$ means the absorptive part of $G^{(4)}(k,p)$ in the variable $(p-k)^2$. We choose the same renormalization condition for A_σ and $\Gamma_{O_{\sigma-1}}$:

$$A_\sigma \Big|_{p^2=-4\pi\mu^2} = \Gamma_{O_{\sigma-1}} \Big|_{p^2=-4\pi\mu^2} = 1. \tag{A.4}$$

The analytic structure of the integrand in eq.A.2, in the k^--plane, is shown in fig.6 where the cuts of $G^{(2)}$, $(k_- \geq \frac{k_\perp^2 - i\varepsilon}{k_+})$, respectively $G^{(4)}$, $(k_- \geq p_- - \frac{k_\perp^2 - i\varepsilon}{(p-k)_+})$, are denoted C and D, respectively A and B.

In relation A.2 the integration path in the k_- plane is along the real axis. We separate the integration region for k^+ into four subregions :

(AC) $(p-k)_+ > 0$, $k_+ < 0$

(AD) $(p-k)_+ > 0$, $k_+ > 0$

(BC) $(p-k)_+ < 0$, $k_+ < 0$

(BD) $(p-k)_+ < 0$, $k_+ > 0$.

For the region (AC) or (BD) we close the k_- contour below or above the real axis and assuming that there are no other singularities in the k_- plane except those of fig.6, the contribution of these regions vanishes. Since for space-like momentum P we choose $P_+ > 0$, $P_- < 0$ and $P_\perp^2 = 0$, it follows that the BC region is empty for the space-like case. So finally the region AD remains and we close the contour around the A-cut so we obtain the discontinuity of $G^{(4)}$ in the $(p-k)_-$ or $(p-k)^2$ variable with the additional constraint $k_+ > 0$.

But this is exactly the integral we had to calculate for the space-like cut vertex A.3. The contribution of the contour at infinity in k_- is substracted after the renormalization condition is imposed.

As a check of the identity of space-like cut vertices and matrix elements of minimal twist Wilson operators we present in table I the results of the two loop calculation in $[\phi^3]_6$ for the anomalous dimensions γ_{A_σ} and $\gamma_{O_{\sigma-1}}$.

Table Iα-β shows this identity diagram by diagram. This fact combined with the renormalization condition A.4 implies that $\Gamma_{O_{\sigma-1}}$ are equal up to two loops.

It is possible to calculate directly the inverse Mellin transform of the anomalous dimension γ_σ, i.e. the function $P(x,\bar{\alpha})$, if we follow the definition in rel.(3.1). This result is given in table IIα-β. In table III the one loop coefficient functions for the current $J(x) = N[\varphi^2(x)]$ and for the space-like case are given as well as their inverse Mellin transform $e(x,\bar{\alpha})$.

APPENDIX B

In this appendix we determine the renormalization conditions for the time-like cut vertices which ensure the validity of the G-L reciprocity relation to all orders in α. The G-L relation at the level of moments for $[\phi^3]_6$ is the following (sec 5.) :

$$\bar{\gamma}_\sigma = \gamma_{o-3} \cdot \tag{B.1}$$

Let $\bar{A}_{\sigma,reg}(p^2,\alpha_0,\varepsilon)$ the dimensionally regularized time-like cut vertices (TLCV) dimension = $6-2\varepsilon$). The renormalized TLCV are defined by the relation :

$$\bar{A}_\sigma\ (p^2/\mu^2,\alpha) = \lim_{\varepsilon\to 0} [Z_\sigma^{-1}\ \bar{A}_{\sigma,reg}(p^2,\alpha_0,\varepsilon)] \tag{B.2}$$

and the renormalization constants Z_σ are determined by a renormalization condition :

$$Z_\sigma(\alpha,\varepsilon)\ .\ \bar{A}_\sigma^0 = \bar{A}_{\sigma,reg}(p^2,\alpha_0,\varepsilon)]_{p^2=4\pi\mu^2} \quad , \tag{B.3}$$

where α_0 the bare coupling constant, $\bar{A}_\sigma^0$ an arbitrary ($\neq 0$) analytic function of the renormalized coupling constant α such that, $\lim_{\alpha\to 0} \bar{A}_\sigma^0 = 1$. We have chosen as substraction point $p^2 = 4\pi\mu^2 > 0$.

The anomalous dimensions for the cut vertices $\bar{A}_\sigma$, corresponding to the renormalization condition (B.3) is defined by :

$$\bar{\gamma}_\sigma(\alpha) \equiv \lim_{\varepsilon\to 0} \bar{\gamma}_\sigma(\alpha,\varepsilon) \ , \tag{B.4a}$$

where

$$\bar{\gamma}_\sigma(\alpha,\varepsilon) \equiv \mu^2 \frac{\partial}{\partial \mu^2} \log Z_\sigma(\alpha,\varepsilon)\Big|_{\alpha_0,\varepsilon,\text{fixed}} = \alpha\beta(\alpha,\varepsilon) \frac{\partial}{\partial\alpha} \log Z_\sigma(\alpha,\varepsilon)\Big|_{\alpha_0,\varepsilon,\text{fixed}} . \tag{B.4b}$$

Under the constraint $Z_{\sigma\ \alpha\to 0} 1$ which follows from (B.3) if we choose $\bar{A}_\sigma^0$ such that, $\lim_{g\to 0} \bar{A}_\sigma^0 = 1$, relation B.4b is equivalent with the following one [19] :

$$Z_\sigma(\alpha,\varepsilon) = e^{\int_0^\alpha \frac{d\alpha'}{\alpha'} \frac{\bar{\gamma}_\sigma(\alpha',\varepsilon)}{b(\alpha',\varepsilon)}}, \tag{B.5}$$

where,

$$\alpha\beta(\alpha,\varepsilon) = \mu^2 \frac{\partial}{\partial \mu^2} \alpha \Big|_{\alpha_0,\varepsilon,\text{fixed}}. \tag{B.6}$$

For two different renormalization conditions on the TLCV which correspond to two different A_σ^0 (A_σ^0 and $A_\sigma'^0$) in eq. B.3 the ratio $A_\sigma'^0/A_\sigma^0$ is related to the corresponding different anomalous dimensions $\bar{\gamma}_\sigma$ and $\bar{\gamma}_\sigma'$ as follows :

$$\bar{A}_\sigma'^0/\bar{A}_\sigma^0 = \lim_{\varepsilon\to 0} \exp\left\{ \int_0^\alpha \frac{d\alpha'}{\alpha'} \left[\bar{\gamma}_\sigma(\alpha',\varepsilon) - \bar{\gamma}_\sigma'(\alpha,\varepsilon)\right]/\beta(\alpha',\varepsilon)\right\}. \tag{B.7}$$

So if we calculate for $A_\sigma^0=1$ the anomalous dimension $\bar{\gamma}_\sigma$ (as we did in ref. 15 up to two loops) and we want to determine $A_\sigma'^0$ such that $\bar{\gamma}_\sigma' = \gamma_{\sigma-3}$ at any fixed order in α then rel.(B.7) gives the required $A_\sigma'^0$.

One can note that the first term of the perturbation expansion in α of $\beta(\alpha,0)$ is of order α we also find from rel.B.7, the known result, that in the limit $\varepsilon \to 0$, $\bar{\gamma}_\sigma(\alpha,0) = \gamma_\sigma'(\alpha,0) + 0(\alpha^2)$, otherwise the ration $\bar{A}_\sigma'^0/\bar{A}_\sigma^0$ would be either infinite or zero. This constraint is valid in our case because the G-L reciprocity relation is known to hold in the one loop approximation ($\bar{\gamma}_\sigma = \gamma_{\sigma-3} + 0(\alpha^2)$).

Another constraint in the choise of the anomalous dimensions $\bar{\gamma}_\sigma'$ comes in the case of a theory with a fixed point different from zero $\beta(a_*) = 0$, $a_* \neq 0$(*). In this case since $\bar{\gamma}_\sigma(\alpha_*)$ and $\bar{\gamma}_\sigma'(\alpha_*)$ control the asymptotic behaviour of the same physical quantities ($Q^2 \to \infty$ behaviour of moments of annihilation structure functions) they must coincide :

$$\bar{\gamma}_\sigma(\alpha_*) = \bar{\gamma}_\sigma'(\alpha_*). \tag{B.8}$$

In asymptotically free theories without any other small fixed point around zero B.8 is valid because both $\bar{\gamma}_\sigma$ and $\bar{\gamma}_\sigma'$ vanish like $\sim \alpha$ for $\alpha \to 0$. In this case rel.B.7 gives the required $A_\sigma'^0$ as an analytic function in α.

(*) We thank H. Kluberg-Stern for bringing this point to our attention.

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TABLE I β

TABLE II β

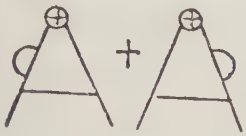
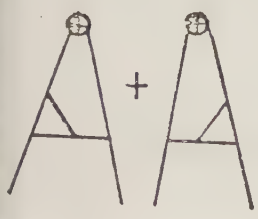
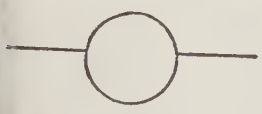
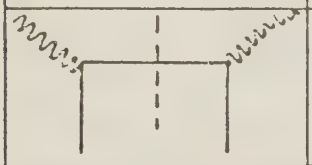
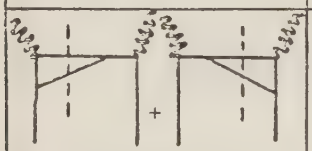
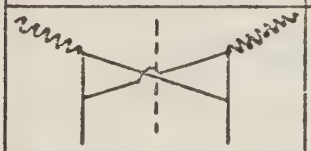
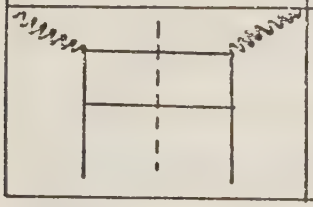
OPERATORS	γ_6	$P(x)$
	$\propto B(2, 6+1)$	$\propto x(1-x)$
	$\frac{\alpha^2}{12} B(2, 6+1)[\psi(1)-\psi(6+3)]$	$\frac{\alpha^2}{12} x(1-x)\log(1-x) - \frac{\alpha^2}{12} x(1-x)$
	$\frac{\alpha^2}{6} B(2, 6+1)[\psi(6+1)-\psi(6+3)]$	$\frac{\alpha^2}{6} x(1-x)\log x$
	$\alpha^2 B(2, 6+1)[\psi(6+3)-\psi(6+1)]$ $-\alpha^2 B(2, 6+1)\ B(1, 6+1)$	$\alpha^2 x(1-x)\log\frac{1}{x}$ $-\alpha^2 [y(1-y) \otimes y]_x$
	$\alpha^2 B(2, 6+1)\ B(2, 6+1)$	$\alpha^2 [y(1-y) \otimes y(1-y)]_x$
	$\alpha^2 B(2, 6+1) B(2, 6+1)[\psi(6+3)-\psi(6+1)]$ $-\alpha^2 B(2, 6+1) \cdot B(2, 6+1)$	$\alpha^2 [y(1-y) \otimes y(1-y)\log\frac{1}{y}]_x$ $-\alpha^2 [y(1-y) \otimes y(1-y)]_x$
	$-\frac{\alpha}{12}$	$-\frac{\alpha}{12} \times \delta(1-x)$
	$-\frac{\alpha^2}{432}$	$-\frac{\alpha^2}{432} \times \delta(1-x)$

TABLE III

	$C_6(1,\alpha)$	$e(x,\alpha)$
	1	$x\delta(1-x)$
	$-\frac{\alpha}{12}+\frac{\alpha}{12}\left(\psi(1)-\psi(6+1)\right)$	$-\frac{\alpha}{12}\cdot x\,\delta(1-x)+\frac{\alpha}{12}\cdot\frac{x}{(1-x)_+}$
	α	$\alpha\cdot x\,\delta(1-x)$
	$-\alpha\,B(1,6+1)$	$-\alpha\cdot x$
	$\alpha\,B(2,6+1)$	$\alpha\cdot x(1-x)$
	$2\alpha B(2,6+1)\left[\psi(6+3)-\psi(6+1)\right]$ $-2\alpha B(2,6+1)$	$2\alpha\cdot x(1-x)\log\frac{1}{x}$ $-2\alpha\cdot x(1-x)$

FIGURES CAPTIONS

Fig. 1 : Forward scattering graph for the process $J(q) + \tilde{\phi}(p) \to J(q) + \tilde{\phi}(p)$.

Fig. 2 : Graph contributing to the cross section for the process $\tilde{\phi}(p) \to \tilde{\phi}(k)$ + anything.

Fig. 3 : Diagram contributing to the radiative corrections of the space-like cut vertex $\theta(k_+)\, k_+^\sigma$.

Fig. 4 : Hand bag diagram of the covariant parton model.

Fig. 5 : Diagram contributing to the radiative corrections of the Wilson operators.

Fig. 6a-6b : Cut structure of the integrand for the graph of Fig. 5.

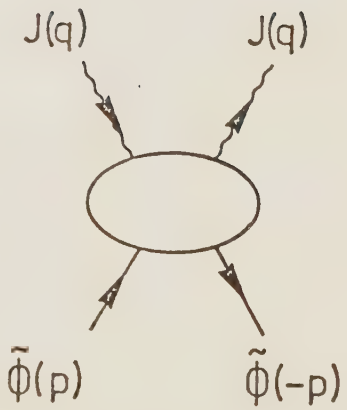


Figure 1a

Figure 1b

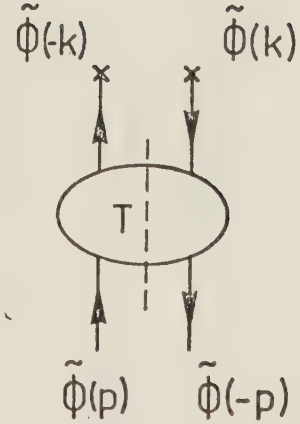


Figure 2a

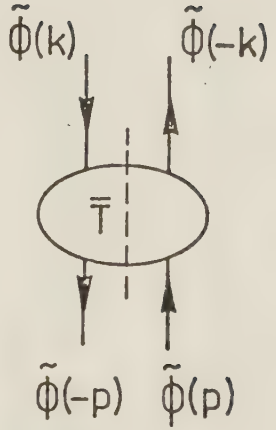


Figure 2b

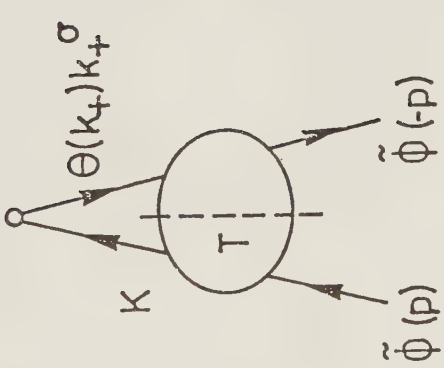


Figure 4

Figure 3

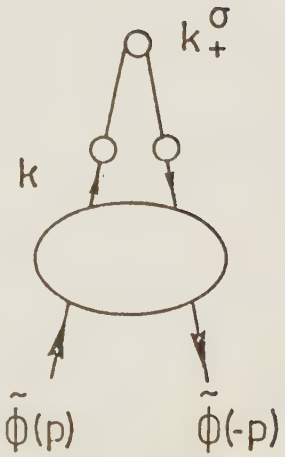


Figure 5

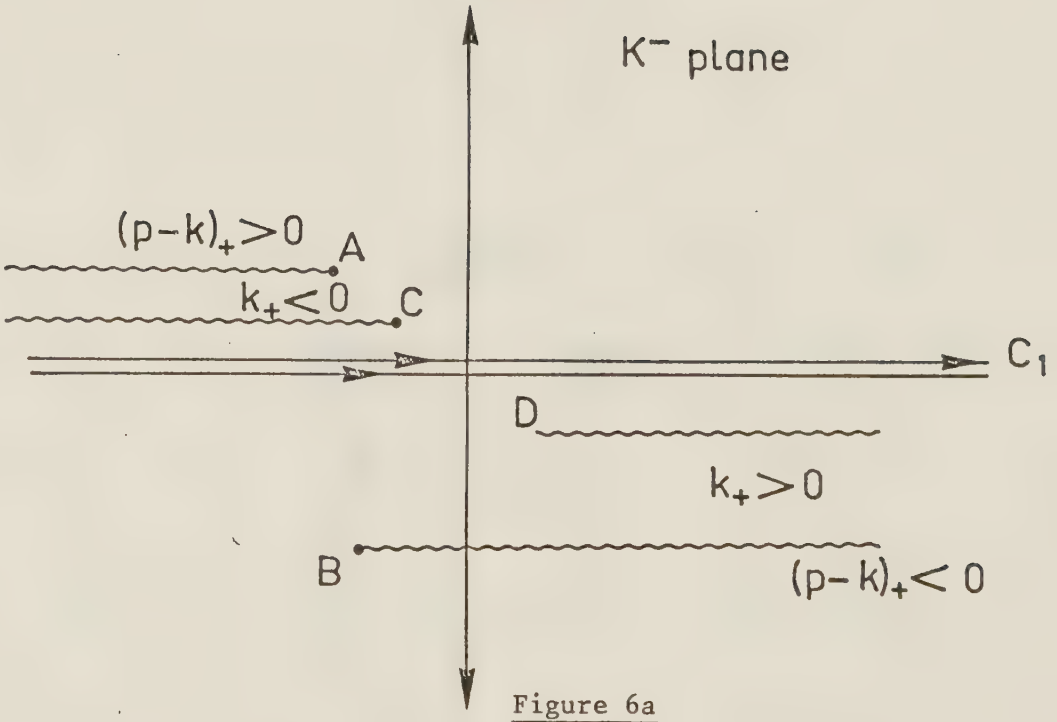


Figure 6a

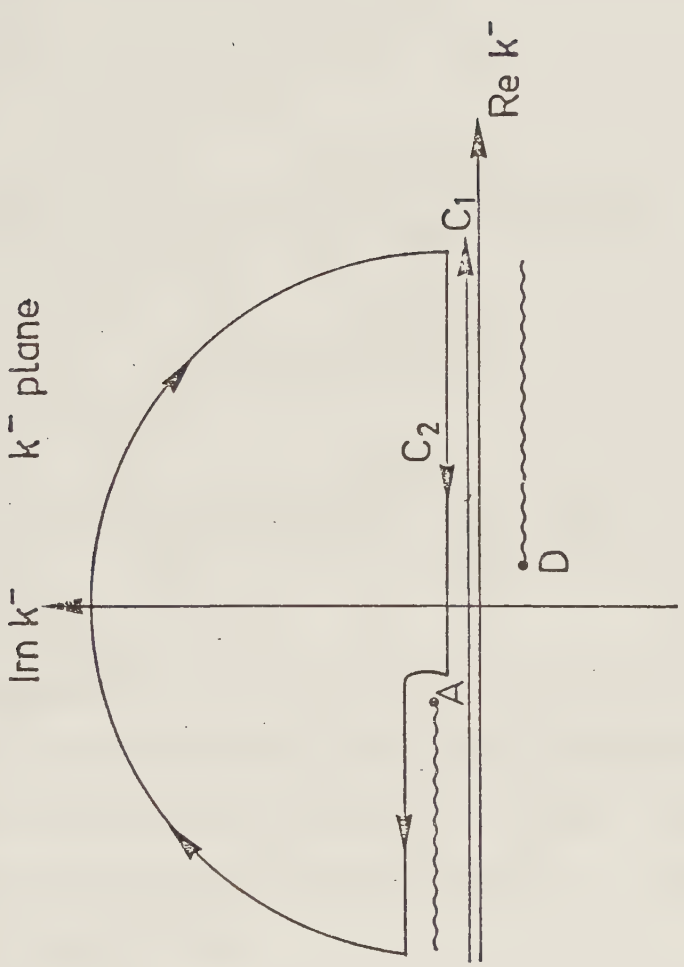


Figure 6.b

HIGHER ORDER CORRECTIONS IN THE CUT-VERTEX FORMALISM

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An algorithm is given for the calculation of higher order corrections to the cut-vertices and their coefficient functions of deep inelastic and annihilation processes in the model field theory $[\phi^3]_6$. The introduction of covariant phase-space variables renders the calculation simple and elucidates the relations between the deep inelastic and annihilation structure functions.

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1. INTRODUCTION

In the past few years, perturbative field theories were used to describe deep inelastic lepton-hadron and totally inclusive electron-positron annihilation processes. Recently, an extension of the light cone expansion method, the so called cut-vertex formalism [1], extended the applicability of these theories to others hard scattering processes. Therefore, we think that it is worth trying to develop simple techniques for the calculation of cut-vertices and the relevant coefficient functions. In this paper we present such a technique.

In Sec. 2 we present our renormalization techniques.

In Sec. 3 we define phase space variables, which are simply related to light cone variables. They are useful in the study of various relations between the space-like and time-like region, while at the same time they simplify the calculation.

In Sec. 4 we give characteristic examples for the calculation of space-like and time-like cut-vertices as well as for their coefficient functions.

In Sec. 5 we present the results of the cut-vertices up to the order g^4 and the coefficient functions up to to the order g^2. We also discuss the relations between the deep inelastic and annihilation structure functions. In this paper, we work in $[\Phi^3]_6$ theory because of the formal simplicity of that theory ; the extension in QCD is postponed to a future paper.

2. RENORMALIZATION ALGORITHM FOR THE CUT VERTICES

In this section we develop the conventions and notations for the renormalization algorithm which is applied to multiplicatively renormalizable Feynman amplitudes in a massless field theory. We will use the dimensional regularization going to $n = 6 - 2\varepsilon$ dimensions, and the renormalization conditions will be taken at a fixed point $4\pi\mu^2$, where μ^2 is the scale appearing in the dimensional regularization.

The prototype theory that we will use is the $[\Phi^3]_6$ which is defined by the bare Lagrangian

$$\mathcal{L}_0 = \frac{1}{2}(\partial_\mu \Phi)(\partial^\mu \Phi) + g_0/3!\,\Phi^3 \tag{2.1}$$

with the following renormalization conditions for the two and three point 1 - P.I Green functions ;

$$\Gamma^{(2)}(P^2)\Big|_{P^2=0} = 0 \tag{2.2a}$$

$$\frac{d}{d(P^2)}\Gamma^{(2)}(P^2)\Big|_{P^2=-4\pi\mu^2} = -1 \tag{2.2b}$$

$$\Gamma^{(3)}(P^2, K^2, (P-K)^2)\Big|_{P^2=K^2=KP=-4\pi\mu^2} = -ig \tag{2.2c}$$

g, is the renormalized coupling constant.

We also define,

$$\chi_0 = g_0^2/(4\pi)^3 \quad , \quad \alpha = g^2/(4\pi)^3 \tag{2.3}$$

Renormalization conditions (2.2a,b,c) determine the anomalous dimension of the field Φ and the β-function,

$$\alpha\beta(\alpha,\varepsilon) = \mu^2 \frac{\partial}{\partial \mu^2} \alpha \Big|_{\varepsilon, \alpha_0 \text{ fixed}} \qquad (2.4)$$

As shown by Mueller, in order to analyse the deep inelastic processes one has to introduce the space-like and time-like cut-vertices (SLCV, TLCV). The SLCV are closely related to the minimal twist Wilson operators, and the TLCV can be formally considered as an "analytic condinuation" of the SLCV These quantities are multiplicatively renormalizable and we give their definition in what follows.

(i) The amputated minimal twist Wilson Operators (W.O) fig.1a

$$\Gamma_{O_\sigma}\left(\frac{P^2}{4\pi\mu^2}, \alpha\right) = \langle 0 | \tilde{\Phi}(P) : \Phi(0) \left(\frac{-i\Delta\cdot\partial}{\Delta\cdot P}\right)^\sigma \Phi(0) : \tilde{\Phi}(P) | 0 \rangle \qquad (2.5a)$$

(ii) The space-like cut-vertex (SLCV) fig. 1 b

$$A^\sigma_{SL}\left(\frac{P^2}{4\pi\mu^2}, \alpha\right) = \int \frac{d^6K}{(2\pi)^6} \left(\frac{\Delta K}{\Delta P}\right)^\sigma \theta(\Delta K)\ T_{SL}(P,K) \qquad (2.5b)$$

(iii) The time-like cut-vertex (TLCV) fig. 1 c

$$A^\sigma_{TL}\left(\frac{P^2}{4\pi\mu^2}, \alpha\right) = \int \frac{d^6K}{(2\pi)^6} \left(\frac{\Delta K}{\Delta P}\right)^{-\sigma} \cdot T_{TL}(P,K) \qquad (2.5c)$$

In the former equations Δ is a light-like vector,

$$\Delta^2 = 0 \ ; \ \Delta P > 0 \qquad (2.6)$$

P^2 is space-like for W.O and SLCV and time-like for TLCV ; σ is any even integer for W.O, and any complex number with $\mathrm{Re}\,\sigma > 0$ for SLCV and TLCV ;

$$T_{\substack{SL \\ TL}}(P,K) = \sum_\lambda \langle 0 | \bar{T}(\tilde{\Phi}(\pm P)\Phi(\mp K)) | \lambda \rangle \langle \lambda | T(\Phi(\pm K)\tilde{\Phi}(\mp P)) | 0 \rangle \left| \mathcal{D}^{(\omega)}(K^2 + i\varepsilon) \right|^2 \qquad (2.7)$$

where $G^{(2)}$ is the renormalized propagator, and $T,\bar{T}$ is the symbol for the time and anti-time ordered product.

Note that the formula(2.5b,c, 2.7) suggest a parton model interpretation of the cut-vertex formalism [4].

We choose the following renormalization conditions for Γ_{O_σ}, A^{σ}_{SL} and A^{σ}_{TL} :

$$\Gamma_{O_\sigma}\left(\frac{P^2}{4\pi\mu^2},\alpha\right)\Big|_{P^2=-4\pi\mu^2} = 1 \tag{2.8a}$$

$$A^{\sigma}_{SL}\left(\frac{P^2}{4\pi\mu^2},\alpha\right)\Big|_{P^2=-4\pi\mu^2} = 1 \tag{2.8b}$$

$$A^{\sigma}_{TL}\left(\frac{P^2}{4\pi\mu^2},\alpha\right)\Big|_{P^2=-4\pi\mu^2} = 1 \tag{2.8c}$$

In the case where O_σ is a conserved current, the renormalization conditions 2.2a,b and 2.8a, valid already in the free field theory, do not spoil the corresponding ward identity. (In $[\phi^3]_6$ theory, there is only one conserved current, the energy momentum tensor O_2). This point is essential for the parton model interpretation of the cut-vertex formalism

Being multiplicatively renormalizable, the W.O, SLCV and TLCV[1] satisfy renormalization group equations (R.G.E). In all that follows we will denote by $A(P^2/4\pi\mu^2, \alpha)$ any of these three quantities. One has,

$$\left(\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha)\frac{\partial}{\partial\alpha} + \gamma_A - 2\gamma_\phi\right)A = 0 \tag{2.11}$$

where γ_A is the "anomalous dimension of A and γ_ϕ the anomalous dimension of the field ϕ. Using eq. 2.11 (R.G.E), one greatly simplifies the computation of A. Indeed, since we work in a massless theory, and since A is function of only one momentum, if is obvious by power counting, that up to the N^{th} order in α, we have

$$A\left(\frac{p^2}{4\pi\mu^2},\alpha\right) = \sum_{m=0}^{N} a_m(\alpha)\,\log^m \frac{4\pi\mu^2}{|p^2|} \qquad (2.12)$$

with

$$a_m(\alpha) = \sum_{k=m}^{N} \alpha^k\, c_{m,k} \qquad (2.13)$$

Therefore by applying the R.G.E (2.11) to the expression (2.12) for A, one gets a recursion relation between the a_m' s which can be expressed as,

$$a_m(\alpha) = \frac{1}{m!}\left(\beta(\alpha)\alpha\frac{\partial}{\partial\alpha} + \gamma_A(\alpha) - 2\gamma_\Phi(\alpha)\right)^m a_0(\alpha) \qquad (2.14)$$

The relation 2.14 shows that, given $\beta(\alpha)$ and $\gamma_\Phi(\alpha)$, A is completely determined by its renormalization condition $a_o(\alpha) \equiv A(1,\alpha)$ and its anomalous dimension $\gamma_A(\alpha)$, or alternatively by $a_o(\alpha)$ and the simple logarithmic coefficient $a_1(\alpha)$, in the expansion 2.12. If $a_o(\alpha) \equiv 1$ as in eqs. (2.8a,b,c) one has simply that

$$\gamma_A - 2\gamma_\Phi = a_1(\alpha) \qquad (2.15)$$

We now turn to the computation technique itself, which will be illustrated up to two loops. First, we must define the wave function and coupling normalization constants Z_Φ and Z_α.

$$\Phi_0 = Z_\Phi^{1/2}(\alpha,\varepsilon)\,\Phi \qquad (2.16)$$

$$\alpha_0 = (4\pi\mu^2)^{\varepsilon} Z_\alpha(\alpha,\varepsilon)\,\alpha \tag{2.17}$$

Z_Φ and Z_α can be computed in perturbation theory from the following conditions

$$\Gamma^{(2)}(p^2, 4\pi\mu^2, \alpha) = \lim_{\varepsilon\to 0} Z_\Phi(\alpha,\varepsilon)\,\Gamma_0^{(2)}(p^2, \alpha_0, \varepsilon) \tag{2.18a}$$

$$\Gamma^{(3)}(p,q, 4\pi\mu^2, \alpha) = \lim_{\varepsilon\to 0} Z_\Phi^{3/2}(\alpha,\varepsilon)\,\Gamma_0^{(3)}(p,q,\alpha_0,\varepsilon) \tag{2.18b}$$

Equation (2.18b) is equivalent to the following one

$$\Gamma^{(3)}(p,q,4\pi\mu^2,\alpha)\Big/_{-ig} = \lim_{\varepsilon\to 0} Z_\Gamma(\alpha,\varepsilon)\,\Gamma_0^{(3)}(p,q,\alpha_0,\varepsilon)\Big/_{-ig_0} \tag{2.18c}$$

with

$$Z_\Gamma(\alpha,\varepsilon) \equiv Z_\Phi^{3/2}(\alpha,\varepsilon)\cdot Z_\alpha^{1/2}(\alpha,\varepsilon) \tag{2.18d}$$

The index "0" denote the bare quantities.

In any finite order in α, say N, Z_Φ and Z_α can be written in any renormalization scheme as :

$$\underset{(\alpha)}{Z_\Phi}(\alpha,\varepsilon) = \sum_{n=0}^{N} \frac{\alpha^n}{\varepsilon^n}\,\underset{(\alpha)}{Z_\Phi^{(n)}}(\varepsilon) \quad ; \quad \underset{(\alpha)}{Z_\Phi^{(0)}}(\varepsilon) = 1 \tag{2.19a}$$

$\underset{(\alpha)}{Z_\Phi^{(n)}}$ is a polynomial in ε of a maximal degree n. On the other hand the renormalizability of A implies the relation ,

$$A\left(\frac{p^2}{4\pi\mu^2}, \alpha\right) = \lim_{\varepsilon\to 0} Z_A(\alpha,\varepsilon)\,A_0(p^2,\alpha_0,\varepsilon) \tag{2.20}$$

In this equation the renormalization constant Z_A has a similar expansion as Z_{Φ} and Z_{α} ,

$$Z_A(\alpha,\varepsilon) = \sum_{n=0}^{N} \frac{\alpha^n}{\varepsilon^n} \; Z_A^{(n)}(\varepsilon) \;\; ; \;\; Z_A^{(0)}(\varepsilon) = 1 \tag{2.19b}$$

We observe, using dimensional analysis that the general from the bare amplitude $A_o(P^2, \alpha_o, \varepsilon)$ is

$$A_o(P^2, \alpha_o, \varepsilon) = 1 + \sum_{k=1}^{N} \frac{\alpha_o^k}{\varepsilon^k} \cdot (P^2)^{-k\varepsilon} \, A^{(k)}(\varepsilon). \tag{2.21}$$

In Eq. 2.21, the $A^{(k)}(\varepsilon)$ are analytic functions of ε. (We will take the convention $P^2 = -p^2$ for the W.O and $P^2 = |p^2|$ for the SLCV and TLCV). Then we combine Eqs. (2.21, 2.17, 2.19a,b) to get :

$$A(t,\alpha) = \lim_{\varepsilon \to 0} \; 1 + \sum_{n=1}^{N} \frac{\alpha^n}{\varepsilon^n} \left(Z_A^{(n)}(\varepsilon) + \sum_{k=0}^{n-1} e^{\varepsilon(n-k)t} \cdot A^{(n-k)}(\varepsilon) \cdot Z_{n-k}^{(k)}(\varepsilon) \right) . \tag{2.22}$$

where we have defined $t = \log \frac{4\pi\mu^2}{p^2}$ and the polynomial in ε, $Z_{n-k}^{(k)}(\varepsilon)$ as

$$Z_A(\alpha,\varepsilon) \left(Z_\alpha(\alpha,\varepsilon) \right)^P = \sum_{k=0}^{N} \frac{\alpha^k}{\varepsilon^k} \; Z_P^{(k)}(\varepsilon) . \tag{2.23}$$

$$P = 0, 1, 2 \ldots N-1$$

In equation 2.22, whatever the renormalization conditions are, the $Z_A^{(n)}$ must satisfy the following constraint coming form the finitness of A as $\varepsilon \to 0$.

$$\frac{d^\lambda}{d\varepsilon^\lambda} \left(Z_A^{(n)}(\varepsilon) + \sum_{k=0}^{n-1} e^{\varepsilon(n-k)t} \, A^{(n-k)}(\varepsilon) \cdot Z_{n-k}^{(k)}(\varepsilon) \right) \Bigg|_{\varepsilon=0} = 0 \tag{2.24}$$

with $\lambda = 0, 1, 2 \ldots n-1$

We illustrate that up to two loops (N = 2)

$$A(t,\alpha) = \lim_{\varepsilon \to 0} \; 1 + \frac{\alpha}{\varepsilon}\left(A^{(1)}(\varepsilon)\,(e^{\varepsilon t} - 1) + A^{(1)}(\varepsilon) + Z_A^{(1)}(\varepsilon) \right)$$
$$+ \frac{\alpha^2}{\varepsilon^2}\left(A^{(2)}(\varepsilon)\,(e^{\varepsilon t} - 1)^2 + \left(2A^{(2)}(\varepsilon) + A^{(1)}(\varepsilon)\,Z^{(1)}(\varepsilon)\right)(e^{\varepsilon t} - 1) \right)$$
$$+ \frac{\alpha^2}{\varepsilon^2}\left(A^{(2)}(\varepsilon) + Z_A^{(2)}(\varepsilon) + Z^{(1)}(\varepsilon)\,A^{(1)}(\varepsilon) \right). \tag{2.25}$$

We note that Eq. 2.23 gives for n = 1

$$Z_1^{(1)}(\varepsilon) = Z_A^{(1)} + Z_\alpha^{(1)} \tag{2.26}$$

Therefore the conditions (2.24) for N = 2 (i.e. the fact that there are no poles in ε in the expression (2.25)) read as

$$A^{(1)}(\varepsilon) + Z_A^{(1)}(\varepsilon)\Big|_{\varepsilon=0} = 0 \tag{2.27a}$$

$$2A^{(2)}(\varepsilon) + A^{(1)}(\varepsilon)\left(Z_A^{(1)}(\varepsilon) + Z_\alpha^{(1)}\right)\Big|_{\varepsilon=0} = 0 \tag{2.27b}$$

$$A^{(2)}(\varepsilon) + A^{(1)}(\varepsilon)\left(Z_A^{(1)}(\varepsilon) + Z_\alpha^{(1)}(\varepsilon)\right) + Z_A^{(2)}(\varepsilon)\Big|_{\varepsilon=0} = 0 \tag{2.27c}$$

$$\frac{d}{d\varepsilon}\left(A^{(2)}(\varepsilon) + A^{(1)}(\varepsilon)\left(Z_A^{(1)}(\varepsilon) + Z_\alpha^{(1)}(\varepsilon)\right) + Z_A^{(2)}(\varepsilon)\right)\Big|_{\varepsilon=0} = 0 \tag{2.27d}$$

Using these constraints and the Eq. (2.25) we obtain for A the following expression valid in any renormalization scheme : $\left(A' = \frac{d}{dt}A(\varepsilon)\Big|_{\varepsilon=0},\ A'' = \frac{d^2}{dt^2}A(\varepsilon)\Big|_{\varepsilon=0}\right)$

$$A(t,\alpha) = 1 + \alpha\left(A^{(1)}t + (A^{(1)} + Z_A^{(1)})'\right) + \alpha A^{(2)}t^2$$
$$+ \alpha^2 t\left[2A^{(2)\prime} + A^{(1)\prime}(Z_\alpha^{(1)} - A^{(1)}) + A^{(1)}(Z_A^{(1)\prime} + Z_\alpha^{(1)\prime})\right]$$
$$+ \alpha^2 \frac{t}{2}\left[A^{(2)\prime\prime} + Z_A^{(2)\prime\prime} + A^{(1)\prime\prime}(Z_\alpha^{(1)} - A^{(1)}) + 2A^{(1)\prime}(Z_\alpha^{(1)\prime} + Z_A^{(1)\prime})\right] \qquad (2.28)$$

We suppose now that Z_α has already been determined and before specifying the renormalization scheme for A we observe the following well known results.

(i) The leading logarithm coefficient is independent of the renormalization conditions for the Green functions of the theory and for the operators, SLCV and TLCV.

(ii) The coefficient of the simple logarithm at the two loops level depends of the renormalization conditions of the operators through $Z_A^{(1)\prime}$, which appears in the terms of order α; as soon as one fixes the renormalization conditions the $Z_A^{(1)\prime}$ is written as a function of $A^{(1)\prime}$

(iii) $Z^{(2)\prime\prime}$ can also be eliminated from the formula (2.28a) in the same way as $Z_A^{(1)\prime}$ and is written as function of $A^{(2)\prime\prime}$, $A^{(1)}$, $A^{(1)\prime}$ and $Z_\alpha^{(1)}$, $Z_\alpha^{(1)\prime}$ after having fixed the renormalization conditions for A.

So we have only to compute the contribution of the bare diagrams of A $A^{(1)}(0)$, $A^{(1)\prime}(0)$, $A^{(2)}(0)$ and $A^{(2)\prime}(0)$, i.e. the coefficients of simple and double poles in ε of the bare two loop diagrams and the coefficient of simple pole and constant term in ε of the one loop bare diagram.

In order to go further we need to compute Z_α and Z_Φ. Therefore we expand the 1-PI Green's functions $\Gamma_o^{(2)}/-ip^2$ and $\Gamma^{(3)}/ig_o\big|_{p^2=q^2=pq}$ as :

$$\Gamma_o^{(2)}(p^2,\alpha_o)/-ip^2 = 1 + \sum_{k=1}^{N} \frac{\alpha_o^k}{\varepsilon^k}(p^2)^{-k\varepsilon} \cdot D^{(k)}(\varepsilon) \qquad (2.29a)$$

$$\Gamma_0^{3}(P,q,\alpha_0)/_{-ig_0}\Big|_{P^2=q^2=Pq} = 1 + \sum_{k=1}^{N} \frac{\alpha_0^k}{\varepsilon^k} (P^2)^{-k\varepsilon} R^{(k)}(\varepsilon) \qquad (2.29b)$$

These expressions are of the same type to that in eq. 2.21, by changing A $A \to D$ and $A \to R$ respectively. Then one can write the analogous equations to 2.28a for the renormalized $\Gamma^{(2)}/-ip^2$ and $\Gamma^{(3)}/-ig\Big|_{p^2=q^2=pq}$

$$\Gamma^{(2)}(P^2, 4\pi\mu^2, \alpha)/_{-iP^2} = (\text{as eq. 2.21, } A \to D,\ Z_A \to Z_\Phi) \qquad (2.28b)$$

$$\Gamma^{(3)}(P,q,4\pi\mu^2,\alpha)/_{-ig}\Big|_{P^2=q^2=Pq} = (\text{as eq. 2.21, } A \to R,\ Z_A \to Z_\Gamma) \qquad (2.28c)$$

Taking into account the renormalization conditions 2.2a,b,c and 2.8a,b,c we can now eliminate all the Z_A, Z_Φ and Z_α dependence in the quantities A, $\Gamma^{(2)}$ and $\Gamma^{(3)}\Big|_{p^2=q^2=pq}$

$$A(t,\alpha) = 1 + \alpha A^{(1)} t + \alpha^2 A^{(2)} t^2 + \alpha^2 t \left(2A^{(2)} - A^{(1)}A^{(1)} - 2A^{(1)}R^{(1)} + 3A^{(1)}D^{(1)}(1-\varepsilon)\right)'_{\varepsilon=0} \qquad (2.30a)$$

$$\Gamma^{(3)}(P,q,t,\alpha)\Big|_{P^2=q^2=Pq} = -ig\left[1 + \alpha R^{(1)} t + \alpha^2 R^{(2)} t^2 + \alpha^2 t \left(2R^{(2)} - 3R^{(1)}R^{(1)} + 3R^{(1)}D^{(1)}(1-\varepsilon)\right)'_{\varepsilon=0}\right] \qquad (2.30b)$$

$$\Gamma^{(2)}(P^2,t,\alpha) = -iP^2\left[1 + \alpha D^{(1)}(t+1) + \alpha^2 D^{(2)} t^2 + \alpha^2 (t+1)\left(2D^{(2)}(1-2\varepsilon) - 2D^{(1)}D^{(1)}(1-\varepsilon), R^{(1)} + 2D^{(1)}D^{(1)}(1-2\varepsilon)\right)'_{\varepsilon=0}\right] \qquad (2.30c)$$

The expressions 2.30a, b,c give the renormalized quantities as functions of bare ones. (the renormalization conditions are therefore automatically taken into account). Using Eq. 2.11 we can now easily determine the anomalous dimension γ_A and γ_Φ as functions of bare quantities.

$$2\gamma_\Phi = \alpha D^{(1)} + \alpha^2 \left[2D^{(2)}(1-2\varepsilon) - 2D^{(1)}(1-\varepsilon)\left(R^{(1)} - D^{(1)}(1-\varepsilon)\right) \right]'_{\varepsilon=0} \qquad (2.31a)$$

$$-\gamma_A + 2\gamma_\Phi = \alpha A^{(1)} + \alpha^2 \left[2A^{(2)} - A^{(1)}A^{(1)} - 2A^{(1)}R^{(1)} + 3A^{(1)}D^{(1)}(1-\varepsilon) \right]'_{\varepsilon=0} \qquad (2.31b)$$

We are now at the point where the renormalized quantities have been expressed in terms of regularized quantities. The following section will be devoted to the phase space analysis necessary to compute them.

3. PHASE SPACE PARAMETRIZATION

In this section we study the phase space domain which is necessary for the computation of the diagrams contributing to the SLCV, the TLCV and their coefficient functions.

Considering the processes shown in figs. 2 a and 2 b ; we choose a frame where

$$q = (q_0, q_3, \vec{0}) \tag{3.1a}$$

$$p = (p_0, p_3, \vec{0}) \tag{3.1b}$$

- In the S.L case we have (fig. 2 a),

$$q^2 < 0 \; ; \; q_+ = q_0 + q_3 > 0 \, , \, q_- = q_0 - q_3 < 0 \tag{3.2a}$$

$$p^2 < 0 \; ; \; p_+ = p_0 + p_3 > 0 \, , \, p_- = p_0 - p_3 < 0 \tag{3.2b}$$

and the condition that the particles produced in the upper and lower parts of fig. 2 a are real particles implies the following inequalities :

$$(p-k)_+ \geqslant 0 \, , \, (p-k)_- \geqslant 0 \tag{3.3a}$$

$$(k-q)_+ \geqslant 0 \, , \, (k-q)_- \geqslant 0 \tag{3.3b}$$

$$(p-k)_+ (p-k)_- \geqslant (p-k)^2 \geqslant 0 \tag{3.3c}$$

$$(k-q)_+ (k-q)_- \geqslant (k-q)^2 \geqslant 0 \tag{3.3d}$$

- In the T.L. case (fig. 2 b) one has

$$q^2 > 0 \; ; \; q_+ > 0 \, , \, q_- > 0 \tag{3.4a}$$

$$P^2 > 0 \; ; \; P_+ > 0 \, , \, P_- > 0 \tag{3.4b}$$

and also

$$(k-P)_+ \geq 0 \, , \, (k-P)_- \geq 0 \tag{3.5a}$$

$$(q-k)_+ \geq 0 \, , \, (q-k)_- \geq 0 \tag{3.5b}$$

$$(k-P)_+ (k-P)_- \geq (k-P)^2 \geq 0 \tag{3.5c}$$

$$(q-k)_+ (q-k)_- \geq (q-k)^2 \geq 0 \tag{3.5d}$$

- We introduce the following set of dimensionless variables

$$x = \frac{q_+}{P^+} \quad , \quad z = \frac{k^2}{k \cdot P_+} \quad , \quad \rho = \frac{k_+}{P_+} \tag{3.6a}$$

and

$$\beta = \frac{P^2}{k^2} \cdot z \cdot \frac{1-\rho}{1-z} \tag{3.6b}$$

then in terms of these variables the phase space domain for the processes of fig. 2a and 2 b are defined by the following sets of inequalities

(i) In the SL case

$$0 \leq x \leq \rho \leq z \leq 1 \tag{3.7a}$$

$$0 \leq \beta_{min} \leq \beta \leq 1 \tag{3.7b}$$

(ii) In the T.L case

$$\infty \geq x \geq \rho \geq z \geq 1 \tag{3.8a}$$

$$0 \leq \beta_{min} \leq \beta \leq 1 \tag{3.8b}$$

where

$$\beta_{min} = \frac{\rho^2}{q^2} \times \frac{(z-x)(1-\rho)}{(\rho-x)(1-z)} \tag{3.9}$$

We note that the variables ρ, z, β and x can be expressed in terms of Lorentz invariants, since one has

$$K^2 = P^2 \frac{z(1-\rho)}{(1-z)} \cdot \frac{1}{\beta} \tag{3.10a}$$

$$(P-K)^2 = -P^2(1-\rho)\frac{(1-\beta)}{\beta} \tag{3.10b}$$

$$(k-q)^2 = -q^2 \frac{\rho-x}{x} \frac{(\beta-\beta_{min})}{\beta} \tag{3.10c}$$

$$(P-q)^2 = -(q^2 - xP^2)\frac{1-x}{x} \tag{3.10d}$$

- We now consider the phase space domains associated with the SLCV and TLCV shown in figs. 1 b and 1 c .

- For the SLCV we have,

$$\rho^2 < 0 \; ; \quad P_+ > 0 \; , \; P_- < 0 \tag{3.11}$$

and the reality of the produced particles together with the presence of the function $\theta(k^+)$ in the fig. 1 b imply the inequalities

$$(P-K)_+ \geq 0 \ , \ (P-K)_- \geq 0 \tag{3.12a}$$

$$(P-K)_+ (P-K)_- \geq (P-K)^2 \geq 0 \tag{3.12b}$$

$$K^+ \geq 0. \tag{3.12c}$$

- For the TLCV we have

$$P^2 > 0 \ ; \ P_+ > 0 \ , \ P_- > 0. \tag{3.13}$$

and also

$$(K-P)_+ \geq 0 \ , \ (K-P)_- \geq 0 \tag{3.14a}$$

$$(K-P)_+ (K-P)_- \geq (K-P)^2 \geq 0 \tag{3.14b}$$

(In the TLCV there is no $\theta(k^+)$ into ρ, z and β as in (eq. (3.6a,b)) we obtain the following sets of inequalities :

(i) SLCV

$$0 \leq \rho \leq z \leq 1 \tag{3.15a}$$

$$0 \leq \beta \leq 1 \tag{3.15b}$$

(ii)-TLCV

$$\infty \geq \rho \geq z \geq 1 \tag{3.16a}$$

$$0 \leq \beta \leq 1 \tag{3.16b}$$

- Introducing a light-like vector Δ^μ :

$$\Delta^2 = 0 \quad \text{and} \quad \Delta P = P_+ > 0 \tag{3.17}$$

we can express k^2, $(P-k)^2$ and $\Delta.k$ as functions of ρ, z and β

$$k^2 = P^2 z \frac{(1-\rho)}{(1-z)} \cdot \frac{1}{\beta} \tag{3.18a}$$

$$(P-k)^2 = -P^2 (1-\rho) \frac{(1-\beta)}{\beta} \tag{3.18b}$$

$$\frac{\Delta k}{\Delta P} = \rho \tag{3.18c}$$

These equations show that z, ρ and β are invariant under a Lorentz transformations.

- For the calculation of the TL cut diagrams we shall use the variables $\bar{x}$, $\bar{\rho}$, $\bar{z}$ and $\bar{\beta}$,

$$\bar{x} = \frac{1}{x}, \quad \bar{\rho} = \frac{1}{\rho}, \quad \bar{z} = \frac{1}{z}, \quad \bar{\beta} = \beta \tag{3.19}$$

rather than x, ρ, z and β. It is worthnoticing that the inequalities (3.8a, b and 3.16a,b) written in terms of these variables,

$$0 \leq \bar{x} \leq \bar{\rho} \leq \bar{z} \leq 1 \tag{3.20a}$$

$$0 \leq \beta_{min} \leq \bar{\beta} \leq 1 \tag{3.20b}$$

are the same as the inequalities (3.7a,b and 3.15a,b) corresponding to the S.L case. We think that this symmetry between the T.L and S.L phase spaces is a key point for the existence of crossing relations between space and time like quantities.

- We will now express the measure $d^n k$ $(n = 6 - 2\varepsilon)$ in terms of the variables

ρ, z, β or $\bar{\rho}$, $\bar{z}$ and $\bar{\beta}$.

First we write,

$$d^{\eta}k = \frac{1}{2} dk_+ dk_- d^{1-2\varepsilon}k_\perp \tag{3.21}$$

and we find

(i) In the S.L case

$$\left(\frac{2\pi}{-p^2}\right)\cdot\int \frac{d^{\eta}k\,(\mu^2)^{\varepsilon}}{(2\pi)^{\eta}(k^2)^{2}} = \frac{e^{\varepsilon t}}{(4\pi)^{3}\Gamma(2-\varepsilon)}\cdot\int_{x\geq 0}^{1} d\rho \int_{z}^{1}\frac{dz}{z}\, z^{-\varepsilon}(1-\rho/z)^{1-\varepsilon}(1-\rho)^{1-\varepsilon}(1-z)^{-1+2\varepsilon}\int_{\beta_{min}\geq 0}^{1}\frac{d\beta}{\beta^{2-\varepsilon}} \tag{3.22a}$$

(ii) In the T.L case

$$\left(\frac{2\pi}{p^2}\right)\cdot\int \frac{d^{\eta}k\,(\mu^2)^{\varepsilon}}{(2\pi)^{\eta}(k^2)^{2}} = \frac{e^{\varepsilon t}}{(4\pi)^{3}\Gamma(2-\varepsilon)}\int_{\bar{x}\geq 0}^{1}\frac{d\bar{\rho}}{\bar{\rho}^{2}}\int_{\bar{z}}^{1}\frac{d\bar{z}}{\bar{z}}\,\bar{z}^{\varepsilon}\left(\frac{\bar{z}}{\bar{\rho}}-1\right)^{1-\varepsilon}\cdot\left(\frac{1-\bar{\rho}}{\bar{\rho}}\right)^{1-\varepsilon}\cdot\left(\frac{1-\bar{z}}{\bar{z}}\right)^{-1+\varepsilon}\int_{\beta_{min}\geq 0}^{1}\frac{d\bar{\beta}}{\bar{\beta}^{2-\varepsilon}} \tag{3.22b}$$

where $t \equiv \log\frac{4\pi\mu^2}{|p^2|}$

- The expressions (3.22a) and (3.22b) allow us to compute in a systematic and straightforward way the bare cut diagrams up to two loops. We will also use the following identities [3] :

$$x^{-1+\varepsilon} = \frac{1}{\varepsilon}\left(\delta(x) + \varepsilon\frac{1}{x_+} + \mathcal{O}(\varepsilon^2)\right) \tag{3.23a}$$

$$\text{disc}\ \frac{1}{(-x-i0)^{1-\varepsilon}} = 2\pi i\left(\delta(x) + \varepsilon\frac{1}{x_+} + \mathcal{O}(\varepsilon^2)\right) \tag{3.23b}$$

and

$$(x+i0)^{\varepsilon} = x^{\varepsilon}\theta(x) + e^{i\pi\varepsilon}(-x)^{\varepsilon}\theta(-x) \tag{3.23c}$$

where

$$\int_0^1 \frac{dx}{x_+}\,\varphi(x) = \int_0^1 \frac{dx}{x}\,\big(\varphi(x) - \varphi(0)\big)\,. \tag{3.23d}$$

These identities will allow us to compute in a very simple way the Laurent expansion in ε of the diagrams. Explicit examples will be given in Sec. 4.

4. TWO LOOP CALCULATION OF SLCV AND TLCV AND THEIR COEFFICIENT FUNCTIONS

We now describe a technique to compute two loop diagrams contributing to the SLCV, TLCV and their coefficient functions for the case of the current $J(X) = :\Phi^2(X):$. The renormalization algorithm is the one described in Sec. 2.

We show in detail the calculation of the diagrams presented in figs. 3a-e contributing to the SLCV and TLCV, and of the diagrams of figs. 4a - e contributing to the S.L and T.L coefficient functions.

We start with the sum of the diagrams $3h_1$ and $3h_2$ and of their relevant counter terms $3h_1'$ and $3h_2'$

The phase space analysis we gave in Sec. 3 allows us to write directly the value of the sum of the diagrams $3h_1$ and $3h_2$.

$$A_\sigma = \alpha \frac{e^{\varepsilon t}}{\Gamma(2-\varepsilon)} \int_0^1 d\rho\, \rho^\sigma \int_\rho^1 \frac{dz}{z} z^{-\varepsilon} (1-\rho/z)^{3-2\varepsilon} (1-\rho)^{3-2\varepsilon} (1-z)^{3+1-3\varepsilon} \int_0^1 \frac{d\beta}{\beta^{2-2\varepsilon}} G(P,K) \qquad (4.1)$$

where the variables ρ, z and β are defined in eqs. (3.2, 3.5 or 3.17a,b,c),

$$t = \log \frac{4\pi\mu^2}{|P^2|}$$

and

$$G(P,K) = \frac{|P^2|}{2\pi} \cdot \frac{\Gamma_+^{(3)} + \Gamma_-^{(3)}}{-ig_0} \cdot \mathrm{disc} \frac{i}{(P-K)^2+i0} = \frac{\Gamma_+^{(3)} + \Gamma_-^{(3)}}{-ig_0} \delta\left(\frac{(P-K)^2}{|P^2|}\right) \qquad (4.2)$$

where $\Gamma_\pm^{(3)}$ is the one loop bare three point function (fig. 3e)

$$\frac{1}{-ig_0}\Gamma_{\pm}^{(3)} = (-ig_0)^2 \int \frac{d^n q}{(2\pi)^n} \frac{i^3}{(q^2 \pm i0)((p-q)^2 \pm i0)((q-K)^2 \pm i0)}$$

$$= \propto \frac{e^{\varepsilon t}\Gamma(1+\varepsilon)}{\varepsilon} \int_0^1 dx \int_0^1 dy\, y^{1-\varepsilon}\left[xy(1-x)\frac{(p-K)^2}{|p^2|} + (1-y)\left(x\frac{-p^2}{|p^2|} + (1-x)\frac{-K^2}{|p^2|} \mp i0\right)\right]^{-\varepsilon} \tag{4.3}$$

(x and y are some Feynman parameters).

Using the constraint $(p-K)^2 = 0$ we have

$$\left.\frac{1}{-ig_0}\Gamma_{\pm}^{(3)}\right|_{(p-K)^2=0} = \propto \frac{e^{\varepsilon t}\Gamma(1+\varepsilon)B(2-\varepsilon,1-\varepsilon)}{\varepsilon} \int_0^1 dx \left(x\frac{-p^2}{|p^2|} + (1-x)\frac{-K^2}{|p^2|} \mp i0\right)^{-\varepsilon} \tag{4.4}$$

Taking into account the identity (3.23c), we have

$$\left.\frac{\Gamma_{+}^{(3)} + \Gamma_{-}^{(3)}}{-ig_0}\right|_{\substack{p^2,K^2<0 \\ (p-K)^2=0}} = \propto \frac{e^{\varepsilon t}R^{(1)}(\varepsilon)}{\varepsilon} \int_0^1 dx\; 2\left(x\frac{-p^2}{|p^2|} + (1-x)\frac{-K^2}{|p^2|}\right)^{-\varepsilon} \tag{4.5}$$

where

$$R^{(1)}(\varepsilon) = \Gamma(1+\varepsilon)B(2-\varepsilon,1-\varepsilon) \tag{4.6}$$

Combining eqs. (4.1, 4.2, 4.5, 4.6) one gets :

$$A_\sigma = \propto^2 \frac{e^{2\varepsilon t}R^{(1)}(\varepsilon)}{\varepsilon\cdot\Gamma(2-\varepsilon)} \int_0^1 d\rho\, \rho^\sigma \int_\rho^1 \frac{dz}{z} z^{-2\varepsilon}(1-z/2)^{1-2\varepsilon}(1-\rho)^{-2\varepsilon}(1-z)^{-1+2\varepsilon} \int_0^1 \frac{d\beta}{\beta}\delta(1-\beta) \int_0^1 dx \left(\frac{(1-z)x}{(1-\rho)z} + (1-x)\right)^{-\varepsilon} \tag{4.7}$$

We can expand $(1-Z)^{-1+2\varepsilon}$ in powers of ε (see eq. 3.23a), and the expression (4.7) is written as

$$A_\sigma = 2\alpha^2 \frac{e^{2\varepsilon t} R^{(1)}(\varepsilon)}{2\varepsilon^2 \Gamma(2-\varepsilon)} \int_0^1 d\rho\, \rho^\sigma \left\{ \int_\rho^1 \frac{dz}{z} z^{-2\varepsilon} (1-\rho/z)^{1-\varepsilon} (1-\rho)^{-\varepsilon} \delta(1-z) \int_0^1 dx\, (1-x)^{-\varepsilon} \right.$$

$$\left. + 2\varepsilon \int_\rho^1 \frac{dz}{z} (1-\rho/z) \frac{1}{(1-z)_+} \int_0^1 dx + \mathcal{O}(\varepsilon^2) \right\}$$

(4.8a)

or equivalently

$$A_\sigma = 2\alpha^2 \frac{e^{2\varepsilon t} R^{(1)}(\varepsilon)}{2\varepsilon^2 \Gamma(2-\varepsilon)} \int_0^1 d\rho\, \rho^\sigma \left\{ (1-\rho) + \varepsilon \left[(1-\rho)(1-3\log(1-\rho) + 2\left((1-y) \otimes \frac{1}{(1-y)_+}\right)_\rho \right] + \mathcal{O}(\varepsilon^2) \right\}$$

(4.8b)

where the convolution product $\otimes$, is defined as follows

$$\left(\varphi(y) \otimes f(y)\right)_\rho = \int_\rho^1 \frac{dy}{y} \varphi(y) f(\rho/y) = \int_\rho^1 \frac{dy}{y} \varphi(\rho/y) f(y) \qquad (4.9)$$

Note that the inverse Mellin transform of the diagrams appears directly in Eq. (4.8b). We now compute the contribution of the diagrams which contain the three point function counter terms $Z_\Gamma^{(1)}$. Since we have choosen the renormalization condition 2.2 c for the three point function we have that (eq. 4.4)

$$Z_\Gamma^{(1)}(\varepsilon) = -R^{(1)}(\varepsilon) = \Gamma(1+\varepsilon)\, B(2-\varepsilon, 1-\varepsilon) \qquad (4.10)$$

Proceeding exactly in the same way as for eq. 4.8a, we obtain :

$$-2\alpha^2 \frac{e^{\varepsilon t} R^{(1)}(\varepsilon)}{\varepsilon^2 \Gamma(2-\varepsilon)} \int_0^1 d\rho\, \rho^\sigma \left\{ (1-\rho) + \varepsilon \left[-2(1-\rho)\log(1-\rho) + \left((1-y) \otimes \frac{1}{(1-y)_+}\right)_\rho \right] + \mathcal{O}(\varepsilon^2) \right\}$$

(4.11)

In eqs. (4.8b) and (4.11) we extract $A^{(2)\sigma}$ and $-A^{(1)\sigma}.R^{(1)}$

$$A_\sigma^{(2)} = 2\frac{R^{(1)}(\varepsilon)}{2\Gamma(2-\varepsilon)}\int_0^1 d\rho\,\rho^\sigma\left[(1-\rho)+\varepsilon\left[(1-\rho)(1-3\log(1-\rho)+(1-y)\otimes\frac{1}{(1-y)_+}\right]+\mathcal{O}(\varepsilon^2)\right] \tag{4.12a}$$

$$-A_\sigma^{(1)}\cdot R^{(1)} = 2\frac{R^{(1)}(\varepsilon)}{\Gamma(2-\varepsilon)}\int_0^1 d\rho\,\rho^\sigma\left[(1-\rho)+\varepsilon\left[-2(1-\rho)\log(1-\rho)+(1-y)\otimes\frac{1}{(1-y)_+}\right]+\mathcal{O}(\varepsilon^2)\right] \tag{4.12b}$$

Then the contribution to the anomalous dimension γ^σ is (see eq. 2.31b),

$$\gamma^\sigma = \alpha^2\left(2A_\sigma^{(2)} - A_\sigma^{(1)}\cdot R^{(1)}\right)'_{\varepsilon=0} \tag{4.13a}$$

$$\gamma^\sigma = \alpha^2\int_0^1 d\rho\,\rho^\sigma(1-\rho)\log\tfrac{1}{\rho} \tag{4.13b}$$

The inverse Mellin transform of γ^σ, P(ρ) is

$$P(\rho) = \alpha^2\rho(1-\rho)\log\tfrac{1}{\rho} \tag{4.14}$$

We now turn to the TLCV, with the example of diagrams $3h_1$, $3h_2$. The computation is very similar to the case of the SLCV, and we will only outline the differences which are of interest for the investigation of the crossing relations between SLCV and TLCV.

Indeed from the phase space analysis of Sec. 3, the sum of these diagrams is given as

$$A_\sigma = \alpha\frac{e^{\varepsilon t}}{\Gamma(2-\varepsilon)}\int_0^1\frac{d\bar\rho}{\bar\rho^{2}}\,\bar\rho^\sigma\int_{\bar\rho}^1\frac{d\bar z}{\bar z}\,\bar z^{\varepsilon}\left(\frac{\bar z}{\bar\rho}-1\right)^{1-2\varepsilon}\left(\frac{1-\bar\rho}{\bar\rho}\right)^{1+\varepsilon}\left(\frac{1-\bar z}{\bar z}\right)^{-1+2\varepsilon}\int_0^1\frac{d\beta}{\beta^{2-\varepsilon}}\,\bar G(\rho,k). \tag{4.15}$$

where

$$\overline{\tilde{\alpha}}(P,K) = \frac{|P^2|}{2\pi}\,\frac{\Gamma_+^{(3)}+\Gamma_-^{(3)}}{-i\gamma_0}\,\mathrm{disc}\,\frac{i}{(P-K)^2+i0} = \frac{\Gamma_+^{(1)}+\Gamma_-^{(1)}}{-i\gamma_0}\,\delta\left(\frac{(P-K)^2}{|P^2|}\right) \qquad (4.16)$$

The expression for the sum $\Gamma_+^{(3)}+\Gamma_-^{(3)}$ becomes after eqs. (4.4, 4.5)

$$\left.\frac{\Gamma_+^{(3)}+\Gamma_-^{(3)}}{-i\gamma_0}\right|_{\substack{P^2,K^2>0\\(P-K)^2=0}} = \alpha\,\frac{e^{\varepsilon t}R^{(3)}}{\varepsilon}\int_0^1 dx\, 2\cos\pi\varepsilon\left(x\frac{\Gamma^2}{|P^2|}\right)^{-\varepsilon}\left(x(1-x)\frac{K^2}{|P^2|}\right)^{-\varepsilon} \qquad (4.17)$$

However, since we need only the $1/\varepsilon$ and ε^0 terms in the expression (4.17) we can take $\cos\varepsilon\pi = 1$.

The rest of the computation of TLCV is done using the same procedure as in the space like case, and the contribution of the diagrams($3h_1$, $3h_2$) and ($3h_1'$,$3h_2'$)to the anomalous dimensions $\tilde{\gamma}^\sigma$ is :

$$\tilde{\gamma}^\sigma = \alpha^2\int_0^1 \frac{d\bar{\rho}}{\bar{\rho}^2}\,\bar{\rho}^\sigma\,\frac{1-\bar{\rho}}{\bar{\rho}}\log\bar{\rho} \qquad (4.18a)$$

$$= \alpha^2\int_1^\infty d\rho\,\rho^{-\sigma}\,(\rho-1)\log\frac{1}{\rho} \qquad (4.19b)$$

and its inverse Mellin transform :

$$\tilde{P}(\rho) = \alpha\,\rho(\rho-1)\log 1/\rho. \qquad (4.19)$$

The relation satisfied by $P(\rho)$ and $\overline{P}(\rho)$

$$P(\rho) = -\tilde{P}(\rho) \qquad (4.20)$$

will be discussed in Section 5.

Let us give also some interesting points appearing in the calculation of the diagram of Fig. 3 e . Since we work in a massless theory the particular

discontinuities of the diagram 3e are without meaning, so one has first to calculate the one loop bare propagator and then take the discontinuity. We find for the regularized bare propagator (fig.3 i)

$$-\alpha \frac{e^{\varepsilon t}}{\varepsilon} D^{(1)}(\varepsilon) \frac{1}{|p^2|} \left(\frac{\beta}{1-\beta}\right)^{1+\varepsilon} \cdot \frac{1}{\left[-(1-\beta) - i0\right]^{1+\varepsilon}} \tag{4.21a}$$

where

$$D^{(1)}(\varepsilon) = -\frac{1}{12}\left(1 + \varepsilon\left(\frac{8}{3} - \gamma\right)\right) \tag{4.21b}$$

Using the distribution identity (3.23b), we find the discontinuity of the regularized one loop bare propagator,

$$\frac{|p^2|}{2\pi} \operatorname{disc} G_0^{(2)}\left((p-k)^2 + i0\right) = \alpha \frac{e^{\varepsilon t} D^{(1)}(\varepsilon)}{\varepsilon}\left(\frac{\beta}{1-\beta}\right)^{1+\varepsilon}\left(\delta(1-\beta) - \frac{\varepsilon}{(1-\beta)_+} + \theta(\varepsilon^2)\right) \tag{4.22}$$

Another interesting technical point in the calculation of diagr 3e is the integration over β which leaves us with an integral of the form

$$I = \int_0^1 \frac{d\beta}{\beta^{1-2\varepsilon}} \cdot \frac{1}{(1-\beta)_+} = \frac{1}{2\varepsilon}\int_0^1 d\beta\left[\delta(\beta) + 2\varepsilon \frac{1}{\beta_+(1-\beta)_+} + \theta(\varepsilon^2)\right] \tag{4.23a}$$

$$\text{or} \quad I = \frac{1}{2\varepsilon}\cdot\left[1 + \varepsilon\int_0^1 d\beta\left(\frac{1}{\beta_+} + \frac{1}{(1-\beta)_+}\right) + \theta(\varepsilon^2)\right] = \frac{1}{2\varepsilon}\left(1 + \mathcal{O}(\varepsilon^2)\right) \tag{4.23b}$$

We now compute the coefficient functions of the structure functions W and $\overline{W}$ of the deep inelastic and semi-inclusive annihilation processes

$$J(-q) + \tilde{\Phi}(p) \longrightarrow X \qquad \text{see fig. 5 a}$$

and

$$J(q) \longrightarrow \tilde{\Phi}(p) + X \qquad \text{see fig. 5 b}$$

in the particular case of the current $J(x) = :\bar{\Phi}^2:$

$$W(q,p) \underset{\substack{-q^2\to\infty \\ q\cdot p/2pq \text{ fixed}}}{=} R(q^2) \sum_x \langle 0|\bar{T}(J(-q)\tilde{\Phi}(p))|x\rangle\langle x|T(\tilde{\Phi}(-p)J(q))|0\rangle_{AMP} \tag{4.24a}$$

$$\bar{W}(q,p) \underset{\substack{-q^2\to\infty \\ q\cdot p/2pq \text{ fixed}}}{=} R(q^2) \sum_x \langle 0|\bar{T}(J(q)\tilde{\Phi}(-p))|x\rangle\langle x|T(\tilde{\Phi}(p)J(-q))|0\rangle_{AMP} \tag{4.24b}$$

where $R(q^2)$ is a normalization factor, such that W and $\bar{W}$ are dimensionless and also with anomalous dimension equal to zero,

$$\left(\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha)\alpha\frac{\partial}{\partial\alpha}\right)W = 0 \tag{4.25a}$$

and

$$\left(\mu^2\frac{\partial}{\partial\mu^2} + \beta(\alpha)\alpha\frac{\partial}{\partial\alpha}\right)\bar{W} = 0. \tag{4.25b}$$

Explicitly,

$$R(q^2) = \frac{|q^2|}{2\pi}\exp\left[\int_\alpha^{\bar{\alpha}(q^2)}\frac{d\alpha'}{\alpha'\beta(\alpha')}\left(2\gamma_J(\alpha') - 2\gamma_\Phi(\alpha')\right)\right] \tag{4.26}$$

where $\bar{\alpha}(q^2)$ is define as follows

$$\log\frac{|q^2|}{4\pi\mu^2} = \int_\alpha^{\bar{\alpha}(q^2)}\frac{d\alpha'}{\alpha'\beta(\alpha')} \tag{4.27}$$

γ_J and γ_Φ are the anomalous dimension of the current J and the field operator Φ, respectively.

As has been shown [1] the moments of the structure function are factorized as follows,

$$\int_0^1 dx\, x^{n-1} W(x,q^2,p^2) = C^n\left(\frac{-q^2}{4\pi\mu^2},\alpha\right) A_{SL}^n\left(\frac{-p^2}{4\pi\mu^2},\alpha\right) \tag{4.28a}$$

$$\int_1^\infty dx\, x^{-\sigma-1}\, \bar{W}(x,q^2,P^2) = \bar{C}^\sigma\left(\frac{q^2}{4\pi\mu^2},\alpha\right) A_{TL}^\sigma\left(\frac{P^2}{4\pi\mu^2},\alpha\right) \qquad (4.28b)$$

where C^σ and $\bar{C}^\sigma$ are the SL and TL coefficient function, and A_{SL}^σ, A_{TL}^σ the SLCV and TLCV. Since we have choosen the renormalization conditions (2.8b,c) we obtain :

$$C^\sigma\left(\frac{-q^2}{4\pi\mu^2},\alpha\right) = \int_0^1 dx\, x^{\sigma-1} W(x,q^2,P^2)\Big|_{|P^2|=4\pi\mu^2} \qquad (4.29a)$$

and

$$\bar{C}^\sigma\left(\frac{q^2}{4\pi\mu^2},\alpha\right) = \int_1^\infty dx\, x^{-\sigma-1} \bar{W}(x,q^2,P^2)\Big|_{|P^2|=4\pi\mu^2} \qquad (4.29b)$$

or with inverse Mellin transformation

$$C\left(x,\frac{-q^2}{4\pi\mu^2},\alpha\right) = W(x,q^2,P^2)\Big|_{|P^2|=4\pi\mu^2} \qquad (4.30a)$$

$$\bar{C}\left(x,\frac{q^2}{4\pi\mu^2},\alpha\right) = \bar{W}(x,q^2,P^2)\Big|_{|P^2|=4\pi\mu^2} \qquad (4.30b)$$

For a next to leading logarithm analysis of the scaling violation in W and $\bar{W}$ one has to compute the coefficient functions $C(x,1,\alpha)$ and $\bar{C}(x,1,\alpha)$ up to one loop. So one has to compute the one loop diagrams shown in figs.(4a-e) in the Bjorken limit, and after that putting $|P^2| = |q^2| = 4\pi\mu^2$. We define the effective charges $e(x,\bar{\alpha}(q^2))$ and $\bar{e}(x,\bar{\alpha}(q^2))$,

$$e(x,\bar{\alpha}(q^2)) \equiv C(x,1,\bar{\alpha}(q^2)) \qquad (4.31a)$$

$$\bar{e}(x,\bar{\alpha}(q^2)) \equiv \bar{C}(x,1,\bar{\alpha}(q^2)) \qquad (4.31b)$$

and using rels (4.30a,b) we have that

$$\mathcal{C}(x,\bar{x}(q^2)) = W\left(\frac{q^2}{4m\mu^2}, \frac{-p^2}{4m\mu^2}, \alpha\right)\Big|_{\substack{p^2=q^2=-4m\mu^2 \\ x=\bar{x}(q^2)}} \tag{4.32a}$$

$$\bar{\mathcal{C}}(x,\bar{x}(q^2)) = \bar{W}\left(\frac{q^2}{4m\mu^2}, \frac{p^2}{4m\mu^2}, \alpha\right)\Big|_{\substack{p^2=q^2=4m\mu^2 \\ \alpha=\bar{x}(q^2)}} \tag{4.32b}$$

We will show the explicit computation of diag. 4d in S.L and T.L case.

Before taking the Bjorken limit the contributions of this diagram to W and $\bar{W}$ are

$$W = \alpha \frac{|q^2|\,(p-q)^2\,\rho_2}{\mathcal{D}^3} \int_\lambda^1 \frac{d\xi\,(1-\xi)(\xi-\lambda)}{\xi^2} \tag{4.33a}$$

$$\bar{W} = \alpha \frac{|q^2|\,(p-q)^2\,\rho_2}{\mathcal{D}^3} \int_\lambda^1 \frac{d\xi\,(1-\xi)(\xi-\lambda)}{\xi^2} \tag{4.33b}$$

where

$$\rho_{1,2} = \frac{1}{2}\left(|2pq| \mp \mathcal{D}\right) \tag{4.34a}$$

$$\mathcal{D} = +\sqrt{(2pq)^2 - 4p^2q^2} \tag{4.34b}$$

$$\lambda = \rho_1/\rho_2 \tag{4.34c}$$

$$\xi = |k^2|/\rho_2 \tag{4.34d}$$

After having done the integration over ξ and neglected the terms of order p^2/q^2 we obtain

$$W = \alpha\, x(1-x)\left[\log \frac{q^2}{p^2} + 2\log\frac{1}{x} - 2\right] \tag{4.35a}$$

$$\overline{W} = \alpha \times (1-x) \left[\log {}^{q^2}/_{p^2} + 2 \log {}^{1}/_{x} - 2 \right] \qquad (4.35b)$$

with

$$\chi = \frac{2|q^2|}{|2pq| + \sqrt{(2pq)^2 - 4p^2q^2}} = \chi_F + \mathcal{O}({}^{p^2}/_{q^2}) \qquad (4.36)$$

We note that among the diagrams (4b - e) only the diagram (4 d) which is non 2PI in the (P - q) channel, contain a log $|P^2|$ term. It is now straightforward to extract from the expressions (4.31a,b, 4.32a,b) the corresponding contribution to the coefficient function and thus to the effective charges.

$$C(x, \frac{-q^2}{4\pi\mu^2}, \alpha) = \alpha \times (1-x) \left[\log \frac{-q^2}{4\pi\mu^2} + 2 \log {}^{1}/_{x} - 2 \right] \qquad (4.37a)$$

$$\overline{C}(x, \frac{q^2}{4\pi\mu^2}, \alpha) = \alpha \times (x-1) \left[\log \frac{q^2}{4\pi\mu^2} + 2 \log {}^{1}/_{x} - 2 \right] \qquad (4.37b)$$

and

$$e(x, \bar{\alpha}(q^2)) = \bar{\alpha}(q^2) \times (1-x) \left(2 \log {}^{1}/_{x} - 2 \right) \qquad (4.38a)$$

$$\bar{e}(x, \bar{\alpha}(q^2)) = \bar{\alpha}(q^2) \times (x-1) \left(2 \log {}^{1}/_{x} - 2 \right) \qquad (4.38b)$$

In the table (I) we have displayed the contribution to e and $\bar{e}$ of all diagrams 4 a - e

5. RESULTS AND RELATION FOR DEEP INELASTIC AND ANNIHILATION STRUCTURE FUNCTIONS

In Ref. [3] we gave a parton model interpretation to the moment equations

$$\int_0^1 dx\, x^{\sigma-1} W(x, P^2, q^2) = A^{\sigma}_{SL}\left(\frac{-P^2}{4\pi\mu^2}, \alpha\right) e^{-\int_{\alpha}^{\bar{\alpha}(q^2)} \frac{d\alpha' \gamma^{\sigma}(\alpha')}{\alpha' \beta(\alpha')}} \cdot C^{\sigma}(1, \bar{\alpha}(q^2)) \tag{5.1}$$

and

$$\int_1^{\infty} dx\, x^{-\sigma-1} \bar{W}(x, P^2, q^2) = A^{\sigma}_{TL}\left(\frac{P^2}{4\pi\mu^2}, \alpha\right) e^{-\int_{\alpha}^{\bar{\alpha}(q^2)} \frac{d\alpha' \bar{\gamma}^{\sigma}(\alpha')}{\alpha' \beta(\alpha')}} \bar{C}^{\sigma}(1, \bar{\alpha}(q^2)) \tag{5.2}$$

as follows. Define the parton density and fragmentation functions such that

$$\int_0^1 dx\, x^{\sigma-1} q(x, q^2) = A^{\sigma}_{SL}\left(\frac{-P^2}{4\pi\mu^2}, \alpha\right) e^{-\int_{x}^{\bar{\alpha}(q^2)} \frac{d\alpha' \gamma^{\sigma}(\alpha')}{\alpha' \beta(\alpha')}} \tag{5.3}$$

and

$$\int_1^{\infty} dx\, x^{-\sigma-1} D(x, q^2) = A^{\sigma}_{TL}\left(\frac{P^2}{4\pi\mu^2}, \alpha\right) e^{-\int_{\alpha}^{\bar{\alpha}(q^2)} \frac{d\alpha' \bar{\gamma}^{\sigma}(\alpha')}{\alpha' \beta(\alpha')}} \tag{5.4}$$

Define also the effective charges $e(X,\bar{\alpha})$ and $\bar{e}(X,\bar{\alpha})$ such that :

$$\int_0^1 dx\, x^{\sigma-1} e(x, \bar{\alpha}(q^2)) = C^{\sigma}(1, \bar{\alpha}(q^2)) \tag{5.5}$$

and

$$\int_1^{\infty} dx\, x^{-\sigma-1} \bar{e}(x, \bar{\alpha}(q^2)) = \bar{C}^{\sigma}(1, \bar{\alpha}(q^2)) \tag{5.6}$$

We can write now the moment eq. 5.1, 5.2 in the form*

$$W(x, P^2, q^2) = e(x, \bar{\alpha}) \underset{S.L}{\otimes} q(x, q^2) \tag{5.7}$$

and

$$\bar{W}(x, P^2, q^2) = \bar{e}(x, \bar{\alpha}) \underset{T_L}{\otimes} D(x, q^2) \tag{5.8}$$

From 5.7 and 5.8 we obtain the following evolution equations for the structure functions W and $\bar{W}$

$$q^2 \frac{\partial}{\partial q^2} W(x, q^2) = \left[e^{-1}(x, \bar{\alpha}) \underset{SL}{\otimes} q^2 \frac{\partial}{\partial q^2} e(x, \bar{\alpha}) + P(x, \bar{\alpha})\right] \otimes W(x, q^2) \tag{5.9}$$

and

$$q^2 \frac{\partial}{\partial q^2} \bar{W}(x, q^2) = \left[\bar{e}^{-1}(x, \bar{\alpha}) \underset{T_L}{\otimes} q^2 \frac{\partial}{\partial q^2} \bar{e}(x, \alpha) + \bar{P}(x, \bar{\alpha})\right] \underset{T_L}{\otimes} \bar{W}(x, q^2) \tag{5.10}$$

where

$$\int_0^1 dx\, x^{\sigma-1} P(x, \bar{\alpha}) = -\gamma^{\sigma}(\bar{\alpha}) \tag{5.11}$$

and similarly for $\bar{P}(\bar{x}, \bar{\alpha})$.

In table II, we give the contribution to the functions P and $\bar{P}$ diagram by diagram up to order α^2. The total contribution for P is :

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*Note that $f(x) \otimes g(x) = \int_x^1 \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$ for the space like region and $f(x) \otimes g(x) = \int_1^x \frac{dy}{y} f\left(\frac{x}{y}\right) g(y)$ for the time like region

$$P(x,\alpha) = \alpha\left(x(1-x) - \frac{1}{12}\delta(1-x)\right) + \alpha^2\left[\frac{1}{2}x(1+x)\log^2 x\right.$$
$$+ \frac{1}{6}x(7-x)\log x + \frac{1}{12}x(1-x)\log(1-x)$$
$$\left. + \frac{11}{12}x(1-x) - \frac{1}{432}x\delta(1-x)\right] \qquad (5.12)$$

and for $\bar{P}$:

$$\bar{P}(x,\alpha) = \alpha\left(x(x-1) - \frac{1}{12}\delta(1-x)\right) + \alpha^2\left[-\frac{1}{2}x(1+x)\log^2 x\right.$$
$$+ \frac{1}{6}x(x-7)\log x + \frac{1}{12}x(x-1)\log(x-1)$$
$$\left. + \frac{11}{12}x(x-1) - \frac{1}{432}x\delta(x-1)\right] \qquad (5.13)$$

In table (I) we give the contribution to the effective charges $e(X,\bar{\alpha})$ and $\bar{e}(x\,\bar{\alpha})$* diagram by diagram up to order α. The total contribution for $e(X,\alpha)$ is

$$e(x,\bar{\alpha}) = x\delta(1-x) + \bar{\alpha}\left[\frac{11}{12}x\delta(1-x) + \frac{1}{12}\frac{x}{(1-x)_+}\right.$$
$$\left. - 2x(1-x)\log x + x(x-2)\right] \qquad (5.14)$$

and for $\bar{e}(x,\alpha)$:

$$\bar{e}(x,\bar{\alpha}) = x\delta(x-1) + \bar{\alpha}\left[\frac{11}{12}x\delta(x-1) + \frac{1}{12}\frac{x}{(x-1)_+}\right.$$
$$\left. - 2x(x-1)\log x + x(2-x)\right] \qquad (5.15)$$

In ref. (5) calculating the renormalization scheme independent quantities $S(x,\bar{\alpha})$ and $\bar{S}(x,\bar{\alpha})$ where

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*For the current $J(x) = :\phi^2(x):$

$$S(x,\bar{\alpha}) = e^{-1}(x,\bar{\alpha}) \otimes q^2 \frac{\partial}{\partial q^2} e(x,\bar{\alpha}) + P(x,\bar{\alpha}) \qquad (5.16)$$

and similarly for $\bar{S}(X,\bar{\alpha})$, we found that in order $\bar{\alpha}^2$,

$$x^3 \bar{S}\left(\frac{1}{x},\bar{\alpha}\right) \neq S(x,\bar{\alpha}) \qquad (5.17)$$

and this implies considering (5.9) and (5.10) that the Gribov-Lipatov relation

$$x^3 \bar{W}\left(\frac{1}{x},q^2\right) = W(x,q^2) \qquad (5.18)$$

is violated in the next to leading logarithmic approximation

Let us turn now to the other interesting possible relation between deep inelastic and annihilation processes, the Drell - Yan relation [7]. We can see from our results that in $[\phi^3]_6$ in the next to leading log approximation the following relation between the structure functions $W(x,q^2)$ and $\bar{W}(x,q^2)$ holds :

$$\bar{W}(x,q^2) = -W(x,q^2) \qquad (5.19)$$

This last equality has to be understood in the following way. It is known that passing from the $0<x<1$ to the $1<x<\infty$ region we meet cuts of the function $W(x,q^2)$ expressing the oppening of thresholds in q^2 for going from negative values to positive ones [8]. Therefore the exact meaning, of the relation (5.19) is the following. As it stands for $x>1$ the right hand side is complex (and doubled values from the appearance of cuts) and the left hand side is real and positive. In order to arrive at the time like structure function $\bar{W}(x,q^2)$, analytically continuing $W(x,q^2)$ in the region $x>1$, we must start from the non forward structure function $W(x_1, q_1, x_2, q_2)$ of the process $J(q_1) + \phi(p_1) \to J(q_2) + \phi(p_2)$ in the space like region

$$q_1^2, q_2^2, p_1^2, p_2^2 < 0 \ , \quad 0 < x_1, x_2 < 1$$

In this region and in the limit $x_1 = x_2 = x$, $q_1^2 = q_2^2 = q^2$, $p_1^2 = p_2^2 = p^2$ the above structure function $W(x_1,q_1, x_2,q_2)$ becomes equal with the structure function $W(x,q)$.

In the time like region $p_1^2, p_2^2, q_1^2, q_2^2 > 0$, $x_1, x_2 > 1$ the function $W(x_1,q_1, x_2,q_2)$ has additional discontinuities in these variables and we must give prescriptions for the values of these variables around their cuts in order to eliminate these discontinuities. In order to recover the annihilation structure function $\bar{W}(x,q)$ starting from the space like one $W(x,q)$ we must give the following definition to $W(x,q)$ [8]

$$W(x,q,p) = \lim_{\varepsilon \to 0} W(x+i\varepsilon, q^2+i\varepsilon, x-i\varepsilon, q^2-i\varepsilon) \tag{5.20}$$

with $p_1^2 = p^2 - i\varepsilon$, $p_2^2 = p^2 + i\varepsilon$,

The right hand side of eq. 5.19 in the physical region for annihilation has to be understood under definition 5.20.

Now we proceed to show that the relation 5.19 is true for $x \neq 1$. For this it is enough to show that the functions $e(x,\bar{\alpha}), \bar{e}(x,\bar{\alpha}), q(x), D(x)$ obey the same type of relations.

$$\bar{e}(x,\alpha) = -e(x,\alpha) \tag{5.21}$$

$$D(x) = -q(x) \tag{5.22}$$

This is true because for $x > 1$.

$$\begin{aligned} W(x,q^2) &= e(x,\bar{\alpha}) \underset{SL}{\otimes} q(x) \\ &= -\bar{e}(x,\bar{\alpha}) \underset{TL}{\otimes} D(x) \\ &= -\bar{W}(x,q^2) \end{aligned} \tag{5.23}$$

That eq. 5.21 is true we find from relation 5.14, 5.15. To prove that relation 5.22 is true one simply has to remember that they are determined by their evolution equations 5.3 and 5.4 :

$$q^2 \frac{\partial}{\partial q^2} q(x,q^2) = P(x,\bar{\alpha}) \underset{SL}{\otimes} q(x,q^2) \tag{5.24a}$$

$$q^2 \frac{\partial}{\partial q^2} D(x,q^2) = \bar{P}(x,\bar{\alpha}) \underset{TL}{\otimes} D(x,q^2) \tag{5.24b}$$

Now for $p^2 = q^2$ we can choose

$$q(x,q^2)\Big|_{p^2=q^2} = \delta(1-x) \tag{5.25a}$$

$$D(x,q^2)\Big|_{p^2=q^2} = \delta(1-x) \tag{5.25b}$$

so for $x \neq 1$ say $x > 1$, we have that

$$q(x,q^2)\Big|_{p^2=q^2} = - D(x,q^2)\Big|_{p^2=q^2} = 0 \tag{5.26}$$

On the other hand from eq. 5.24 a, 5.25 b and the fact that $p(x,\bar{\alpha})$ and $\bar{p}(x,\bar{\alpha})$ satisfy relation 5.19* we find

$$q^2 \frac{\partial}{\partial q^2} q(x,q^2)\Big|_{p^2=q^2} = - q^2 \frac{\partial}{\partial q^2} D(x,q^2)\Big|_{p^2=q^2} \tag{5.27}$$

and so on for higher derivatives. Thus by Taylor expansion of $q(x,q^2)$

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*The procedure we follow to analytically continue the dangerous $\log(1-x)$ terms in 5.12 is the following for $x<1$:

$$\log(1-x) = \frac{1}{2}\left(\log(1-x+i\varepsilon) + \log(1-x-i\varepsilon)\right)$$

$$\longrightarrow \log(x-1) \quad \text{for} \quad x > 1.$$

*See also remark in p. 39.

and $D(x,q^2)$ around $p^2 = q^2$ we obtain the desired result ($x > 1$) :

$$\eta(x,q^2) = -D(x,q^2) \qquad (5.28)$$

CONCLUSION

The calculations have been presented for the cut vertices (space-like and time like) in $[\phi^3]_6$ up to order g^4 and for the coefficient functions of deep inelastic and annihilation processes up to order g^2. The explicit formulation of the dimensional regularization procedure for cut-vertices allows for the complete characterization of the renormalization scheme dependence. Convenient phase space variables render the calculations transparent and they illustrate the relation of time like and space like regions. Explicit examples offer the ground for the demonstration of the necessary computational techniques which we believe simplify enough the calculations.

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FIGURE CAPTIONS

Fig. 1a : Graph for the matrix element of the minimal twist Wilson operators ; $p^2 < 0$, $\Lambda^2 = 0$.

Fig. 1b : Space-like cut vertices ; $p^2 < 0$, $\Lambda^2 = P$, $\Delta p > 0$.

1c : Time-like cut vertices ; $p^2 > 0$, $\Delta^2 = 0$, $\Lambda p < 0$.

Fig. 2a : One virtual particle (k^μ) configuration of the process $J(-q) + \rho(p) \to x$ p^2, q^2, $2pq < 0$.

2b : One virtual particle (k^μ) configuration of the process $J(q) \to \rho(p) + x$; p^2, q^2, $2pq > 0$.

Fig.3a-h : Relevant diagrams for both space-like and time-like cut-vertices, up to the two loop approximation.

3i-k : The two point function diagrams.

3ℓ : The three point function at one loop level.

Fig. 4a-e : Diagrams related to the space like and time like structure functions of the processes $J((q) + \varphi(p) \to x$ and $J(q) \to \varphi(p) + x$, up loop approximation.

Fig.5a : $J(-q) + \varphi(p) \to x$; $q^2, p^2, 2pq < 0$.

5b : $J(q) \to \varphi(p) + x$; $q^2, p^2, 2pq > 0$.

TABLE CAPTIONS

I. Graph by graph contribution to deep inelastic and semi-inclusive annihilation charge densities $e(x,\alpha)$ and $\bar{e}(x,\alpha)$.

II. Graph by graph contribution to the inverse Mellin transform of the anomalous dimensions $p(x)$ and $\bar{p}(x)$ of the space like and time like cut vertices.

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*The distribution $\frac{1}{y_+}$ is defined in general [3] as

$$\int_{-\infty}^{\infty} dy \frac{1}{y_+} \varphi(y) = \int_0^1 dy \frac{\varphi(y)-\varphi(0)}{y} + \int_1^{\infty} dy \frac{\varphi(y)}{y}$$

$$= \lim_{\varepsilon\to 0} \int_{\varepsilon}^{\infty} dy \frac{\varphi(y)}{y} - \int_{\varepsilon}^{1} dy \frac{\varphi(0)}{y}$$

In the space like case the domain of the functions φ is $0 \le x \le 1$ and one has

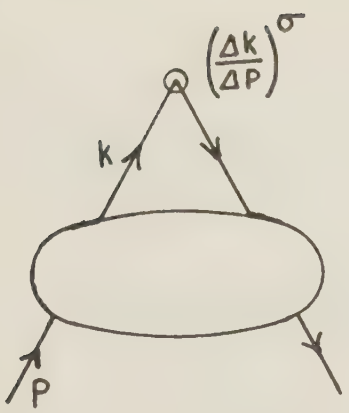
$$\int_0^1 \frac{dx}{(1-x)_+} \varphi(x) = \int_0^1 dx \frac{\varphi(x)-\varphi(1)}{1-x}$$

In the time like case where the corresponding domain is $\infty > x > 1$

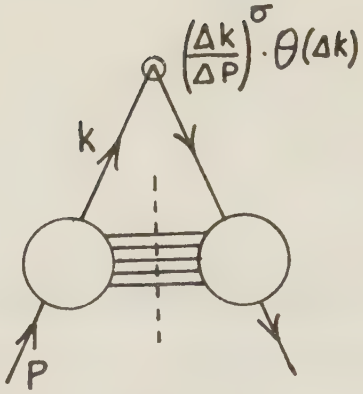
$$\int_1^{\infty} \frac{dx}{(x-1)_+} \varphi(x) = \lim_{\varepsilon\to 0} \int_{1+\varepsilon}^{\infty} \frac{dx\ \varphi(x)}{(x-1)} - \int_{1+\varepsilon}^{2} \frac{dx\ \varphi(1)}{(x-1)}$$

$$= \lim_{\varepsilon\to 0} \int_0^{1-\varepsilon} \frac{d\bar{x}}{1-\bar{x}} \cdot \frac{1}{\bar{x}} \varphi\left(\frac{1}{\bar{x}}\right) - \int_0^{1-\varepsilon} \frac{dt\ \varphi(1)}{1-t}$$

$$= \int_0^1 \frac{d\bar{x}}{(1-\bar{x})_+} \left(\frac{1}{\bar{x}} \varphi\left(\frac{1}{\bar{x}}\right)\right)$$



1 a



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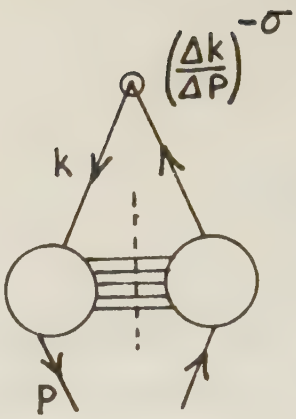
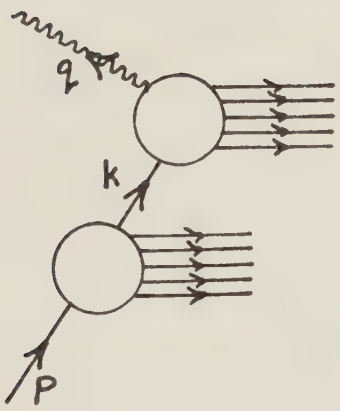
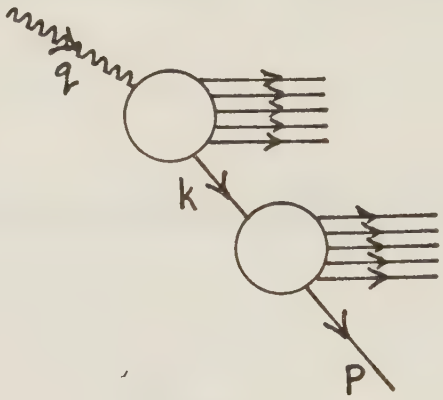


1 c



2 a



2 b

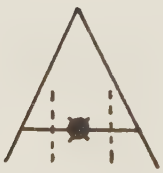
3 a

3 b

3 c

3 d

3 e



3 e'

3 f_1

3 f_2

3 g_1

3 g_2

3 h_1

3 h_2

3 h_1'

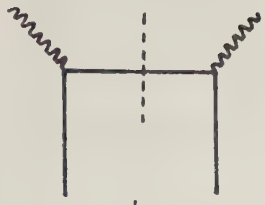
3 h_2'

3 i

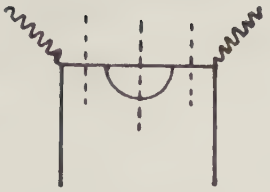
3 j

3 k

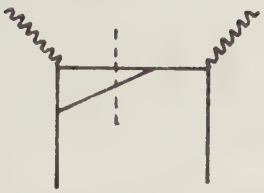
3 ℓ



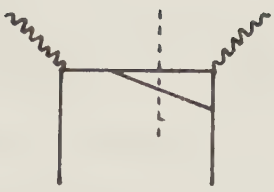
4 a



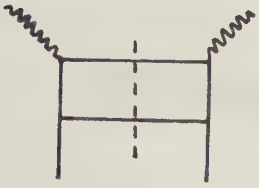
4 b



4 c_1

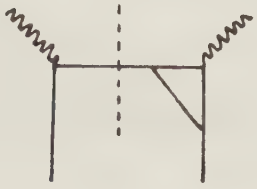


4 c_2

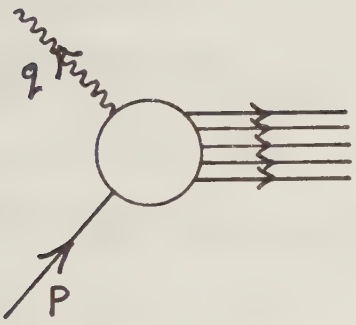


4 d

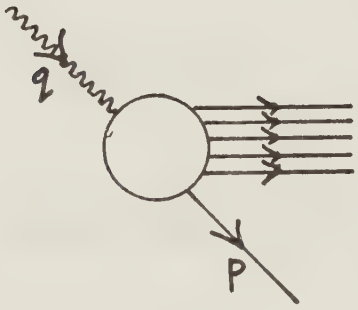
4 e_1



4 e_2



5 a



5 b

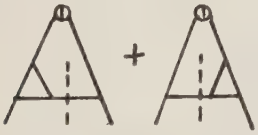
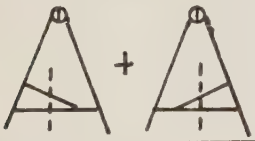
UT-VERTICES	$P(x)$	$\overline{P}(x)$
	$\alpha\, x(1-x)$	$\alpha\, x(x-1)$
	$\frac{\alpha^2}{12} x(1-x)\log(1-x) - \frac{\alpha^2}{12} x(1-x)$	$\frac{\alpha^2}{12} x(x-1)\log(x-1) - \frac{\alpha^2}{12} x(x-1)$
	$\frac{\alpha^2}{6} x(1-x)\log x$	$\frac{\alpha^2}{6} x(x-1)\log x$
	$\alpha^2 x(1-x)\log\frac{1}{x}$	$\alpha^2 x(x-1)\log\frac{1}{x}$
	$\alpha^2 x(1-x) + \alpha^2 x\log x$	$\alpha^2 x(x-1) - \alpha^2 x\log x$
	$\alpha^2 x(1+x)\log\frac{1}{x} - 2\alpha^2 x(1-x)$	$\alpha^2 x(1+x)\log x - 2\alpha^2 x(x-1)$
	$-\alpha^2 x(1+x)\log\frac{1}{x} + 2\alpha^2 x(1-x)$ $+\frac{\alpha^2}{2} x(1+x)\log^2 x + \alpha^2 x(1-x)\log x$	$-\alpha^2 x(1+x)\log x + 2\alpha^2 x(x-1)$ $-\frac{\alpha^2}{2} x(1+x)\log^2 x + \alpha^2 x(x-1)\log x$
	$-\frac{\alpha}{12} x\,\delta(1-x)$	$-\frac{\alpha}{12} x\,\delta(x-1)$
	$-\frac{\alpha^2}{432} x\,\delta(1-x)$	$-\frac{\alpha^2}{432} x\,\delta(x-1)$.

Table 1

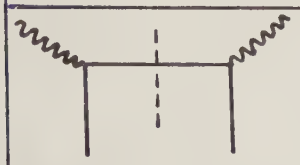
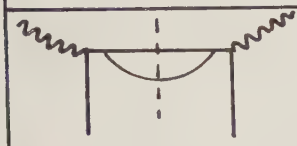
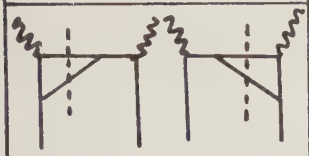
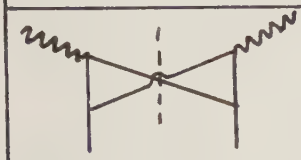
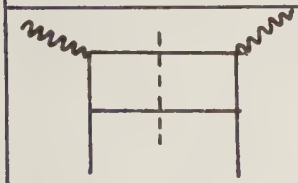
	$e(x,\alpha)$	$\overline{e}(x,\alpha)$
	$x\delta(1-x)$	$x\delta(x-1)$
	$-\frac{\alpha}{12}x\delta(1-x)+\frac{\alpha}{12}\frac{x}{(1-x)_+}$	$-\frac{\alpha}{12}x\delta(x-1)+\frac{\alpha}{12}\frac{x}{(x-1)_+}$ *
	$\alpha\, x\delta(1-x)$	$\alpha\, x\delta(x-1)$
	$-\alpha x$	αx
	$\alpha\, x(1-x)$	$\alpha\, x(x-1)$
	$2\alpha\, x(1-x)\log\frac{1}{x}$ $-2\alpha\, x(1-x)$.	$2\alpha\, x(x-1)\log\frac{1}{x}$ $-2\alpha\, x(x-1)$.

Table 2

- VI -

CORRECTIONS D'ORDRES SUPERIEURS
DANS LES THEORIES DE
JAUGE NON-ABELIENNE

1. Les transitions permises aux ordres supérieurs entre quarks, antiquarks et gluons

Dans le cas scalaire, un seul noyau $P(x,\alpha)$ contrôle la dépendance en Q de la distribution des partons $q(x,Q)$. Par contre, dans les théories de jauge non abéliennes avec f saveurs et N^2-1 gluons, on a en général une matrice $P_{ij}(x,\alpha)$ à $(2f+N^2-1)\times(2f+N^2-1)$ éléments. Les indices ij peuvent être ceux des quarks, antiquarks ou bien des gluons. Le nombre des éléments de la matrice P_{ij} indépendants se trouve très réduit, du fait des symétries. Ce sont: (i) le $SU(N)_c$ de couleur qui est une symétrie exacte, (ii) $SU(f)$ des saveurs, que l'on considère comme exacte dans l'approximation où on néglige les masses des quarks et les effets de "twist" élevé, et (iii) la symétrie de conjugaison de charge qui échange les quarks et les antiquarks.

Au premier ordre de la théorie des perturbations on a seulement quatre éléments de matrice indépendants

$$P^{o}_{qq}(x)\ ,\ P^{o}_{qG}(x)\ ,\ P^{o}_{Gq}(x) \quad \text{et} \quad P^{o}_{GG}(x) \qquad (1)$$

Le premier exprime la variation de la probabilité de production d'un (anti-)quark à partir d'un autre (anti-)quark; le deuxième, celle d'un (anti-) quark à partir d'un gluon; le troisième, celle d'un gluon à partir d'un (anti-) quark et le quatrième, celle d'un gluon à partir d'un autre gluon. Les transitions qui changent la saveur (par exemple $u \longrightarrow d$), sont interdites à l'ordre $\alpha(Q)$; il en est de même des transitions quark $\longrightarrow$ antiquark.

Un effet nouveau est l'apparition de transitionsde ce type aux ordres supérieurs. On trouve alors des éléments supplémentaires non nuls, à l'ordre $\alpha^2(Q)$. Cela justifie le calcul des ordres supérieurs en α, et en premier lieu l'ordre α^2, où le nombre d'éléments indépendants passe de quatre à six.

Les six quantités indépendantes de P_{ij} sont les suivantes : i) P^{NS}_{qq} qui exprime les transitions diagonales non-singulet entre les quarks et les antiquarks, par exemple $u \longrightarrow u$, $\bar{u} \longrightarrow \bar{u}$ (fig. 1a)

(ii) $P_{q\bar{q}}^{NS}$, qui exprime les transitions non diagonales non singulet d'un quark à son antiquark et inversement; par exemple $u \to \bar{u}$, $\bar{u} \to u$. (fig.1b)

(iii) $P_{q_1 q_2}$, qui exprime les transitions singulet, diagonales ou non entre les quarks ou les antiquarks; par exemple $u \longrightarrow u, \bar{u}, d, \bar{d}...$ (fig.1c)

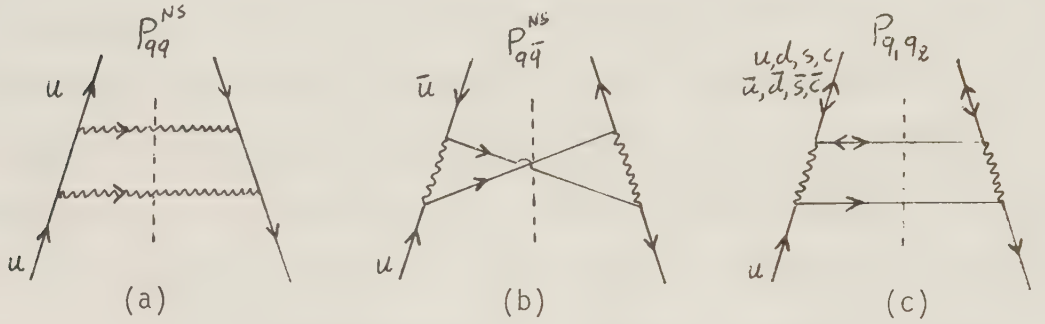


Fig. 1 : Des diagrammes qui contribuent à (a) P_{qq}^{NS} (b) $P_{q\bar{q}}^{NS}$ (c) $P_{q_1 q_2}$

(iv) P_{qG} , qui exprime les transitions d'un gluon aux quarks ou antiquarks; par exemple $G \to u, \bar{u}, d, \bar{d}...$ (fig.2a)

(v) P_{Gq} , qui exprime les transitions des (anti-)quarks aux gluons. (fig.2b)

(vi) P_{GG} , qui exprime les transitions gluon $\longrightarrow$ gluon (fig.2c).

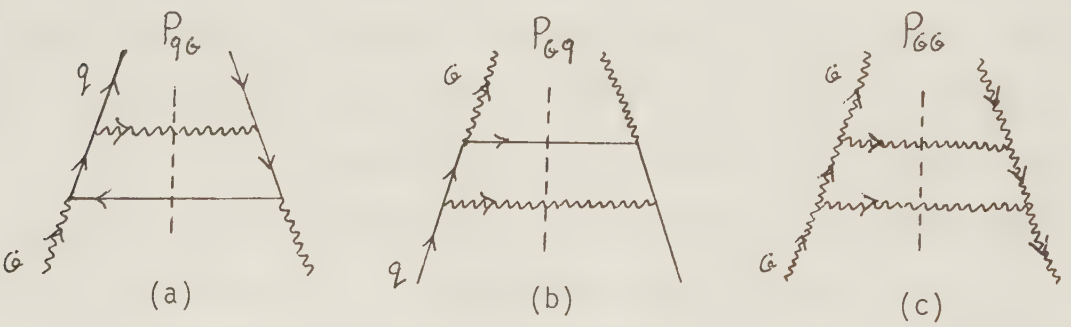


Fig. 2 : Des diagrammes qui contribuent à (a) P_{qG} (b) P_{Gq} (c) P_{GG}

Les équations suivantes contrôlent les variations suivant Q des fonctions de distribution des quarks, antiquarks et gluons.

$$\dot{u} = P_{qq}^{NS} \otimes u + P_{q\bar{q}}^{NS} \otimes \bar{u} + P_{q_1 q_2} \otimes [u + \bar{u} + d + \bar{d} + \cdots] + P_{qG} \otimes G$$
$$\dot{\bar{u}} = P_{qq}^{NS} \otimes \bar{u} + P_{q\bar{q}}^{NS} \otimes u + P_{q_1 q_2} \otimes [u + \bar{u} + d + \bar{d} + \cdots] + P_{qG} \otimes G$$
$$\dot{d} = P_{qq}^{NS} \otimes d + P_{q\bar{q}}^{NS} \otimes \bar{d} + P_{q_1 q_2} \otimes [u + \bar{u} + d + \bar{d} + \cdots] + P_{qG} \otimes G$$
$$\dot{\bar{d}} = P_{qq}^{NS} \otimes \bar{d} + P_{q\bar{q}}^{NS} \otimes d + P_{q_1 q_2} \otimes [u + \bar{u} + d + \bar{d} + \cdots] + P_{qG} \otimes G$$
$$\vdots$$
$$\dot{G} = P_{Gq} \otimes [u + \bar{u} + d + \bar{d} + \cdots] + P_{GG} \otimes G \qquad (2)$$

où $\dot{u} = Q \frac{\partial}{\partial Q} u$ et $P_{qq} \otimes u = \int_0^1 \frac{dy}{y} P_{qq}\left(\frac{x}{y}\right) u(y)$ (3)

La différence $u - \bar{u}$ vérifie une équation diagonale simple

$$(\dot{u} - \dot{\bar{u}}) = \left(P_{qq}^{NS} - P_{q\bar{q}}^{NS}\right) \otimes (u - \bar{u}) \qquad (4)$$

Il en est de même pour toutes les différences $q_i - \bar{q}_i$ (q_i = u,d,s,c).

Il existe aussi d'autres combinaisons des quarks et antiquarks satisfaisant à des équations diagonales, comme la combinaison u + ū - d - d̄

$$(\dot{u} + \dot{\bar{u}} - \dot{d} - \dot{\bar{d}}) = \left(P_{qq}^{NS} + P_{q\bar{q}}^{NS}\right) \otimes (u + \bar{u} - d - \bar{d}) \qquad (5)$$

Cette équation est aussi valable pour toute combinaison de la forme

$q_1 + \bar{q}_1 - q_2 - \bar{q}_2$ ($q_{1,2}$ = u,d,s,c).

La somme des quarks et antiquarks, ainsi que la distribution des gluons, satisfont à des équations d'évolution couplées:

$$\dot{\Sigma} = \left(P_{qq}^{NS} + P_{q\bar{q}}^{NS} + P_{q_1 q_2}\right) \otimes \Sigma + P_{qG} \otimes G \qquad (6a)$$

$$\dot{G} = P_{qG} \otimes \Sigma + P_{GG} \otimes G \qquad (6b)$$

avec $\Sigma(x,Q) = u(x,Q) + \bar{u}(x,Q) + d(x,Q) + \bar{d}(x,Q) + S(x,Q) + \bar{S}(x,Q) + C(x,Q) + \bar{C}(x,Q)$ (7)

On s'intéressera aux quantités suivantes:

$$P_{qq}^{NS}(-) \equiv P_{qq}^{NS} - P_{q\bar{q}}^{NS} \quad , \quad P_{qq}^{NS}(+) \equiv P_{qq}^{NS} + P_{q\bar{q}}^{NS} \qquad (8a)$$

ainsi qu'aux quantités:

$$P_{qq}^{S} \equiv P_{qq}^{NS} + P_{q\bar{q}}^{NS} + P_{q_1 q_2} \;,\; P_{qG} \;,\; P_{Gq} \;,\; P_{GG} \;. \tag{8b}$$

Le fait que l'équation (4) est diagonale découle du comportement de la différence $q_i - \bar{q}_i$ qui est impaire vis à vis des transformations de conjugaison de charge:

$$(q_i - \bar{q}_i) \xrightarrow{C} -(q_i - \bar{q}_i) \tag{9}$$

Ni la densité des gluons G ni la somme Σ ne peuvent alors intervenir, puisque G et Σ sont des fonctions paires, $C=+$ pour la conjugaison de charge. La forme diagonale de l'équation (5) a une autre origine: la combinaison $q_1 + \bar{q}_1 - q_2 - \bar{q}_2$, bien qu'elle se comporte de la même manière que G et Σ pour la conjugaison de charge ($C = +$), ne peut pas être couplée à G et Σ, puisqu'elle est non singulet par rapport aux transformations $SU(f)$ alors que G et Σ sont singuletsde $SU(f)$.

Les quantités G et Σ ont les mêmes nombres quantiques (singulet par rapport à $SU(f)$ avec $C = +$). Elles obéissent à un système d'équations intégrodifférentielles couplées dont le noyau d'évolution est représenté par la matrice:

$$P_{ij} \equiv \begin{pmatrix} P_{qq}^{S} & P_{qG} \\ P_{Gq} & P_{GG} \end{pmatrix} \tag{10}$$

Une fonction de structure ayant une valeur de C bien déterminée s'exprime à l'aide de combinaisons des fonctions de distribution des quarks et des gluons ayant le même C. Il en est de même pour les nombres quantiques du groupe $SU(f)$. Il en résulte que la dépendance en Q de ces fonctions de structure est contrôlée par des noyaux de nombres quantiques bien déterminés. Ce sont: $P_{qq}^{NS}(+)$, $P_{qq}^{NS}(-)$ et P_{ij}.

$F_2^{NS}(x,Q) = \frac{1}{x} F_2^{ep-en}(x,Q)$, par exemple, est une quantité qui a c = +1 et qui est non singulet pour les transformations de $SU(f)$. Elle s'exprime comme un produit de convolution d'une charge effective $e_2^{NS}(x,\alpha)$ et d'une combinaison des densités de quarks et d'antiquarks, $q_+^{NS}(x,Q)$, qui sera non-singulet pour $SU(f)$.

$$F_2^{NS}(x,Q) = e_2^{NS}(x,\alpha(Q)) \otimes q_+^{NS}(x,Q) \qquad (11a)$$

Comme $q_+^{NS}(x,Q)$ satisfait à l'équation

$$\dot{q}_+^{NS}(x,Q) = P_{qq}^{NS}(x,\alpha(Q);+) \otimes q_+^{NS}(x,Q) \qquad (11b)$$

on peut éliminer $q_+(x,Q)$ et $\dot{q}_+(x,Q)$ des équations (11a) et (11b). On trouve alors que la fonction de structure $F_2^{NS}(x,Q)$ et sa dérivée $\dot{F}_2^{NS}(x,Q)$ sont reliées par:

$$\dot{F}_2^{NS}(x,Q) = K_2^{NS}(x,\alpha(Q);+) \otimes F_2^{NS}(x,Q) \qquad (12)$$

où le noyau $K_2^{NS}(x,\alpha;+)$ s'exprime à l'aide de $e_2^{NS}(x,\alpha)$ et $P_{qq}^{NS}(x,\alpha;+)$:(+)

$$K_2^{NS}(x,\alpha(Q);+) = \dot{e}_2^{NS}(x,\alpha(Q)) \otimes e_2^{NS\,-1}(x,\alpha(Q)) + P_{qq}^{NS}(x,\alpha(Q);+) \qquad (13)$$

Dans le cas des processus d'annihilation, les fonctions de fragmentation $\tilde{F}_2^{NS}(x,Q)$ obéissent à des équations analogues à (12,13), que l'on obtient en remplaçant $F_2^{NS}(x,Q)$ par $\tilde{F}_2^{NS}(x,Q)$ et $K_2^{NS}(x,\alpha;+)$ par $\tilde{K}_2^{NS}(x,\alpha;+)$.

La suite de ce chapitre décrit les calculs des charges effectives $e_r^{(\sim)NS}(x,\alpha)$ et des noyaux $\tilde{P}^{(\sim)NS}(x,\alpha;c)$, avec c = $\pm$ 1, dans l'approximation des diagrammes à deux boucles ($\alpha^2(Q)$).

(+) L'inverse $A^{-1}(x)$, au sens des produits de convolution, est défini par l'équation

$$A^{-1}(x) \otimes A(x) = \delta(1-x)$$

SPACE AND TIME LIKE CUT VERTICES IN QCD BEYOND THE LEADING ORDER

(I) NON SINGLET SECTOR

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ABSTRACT

In the framework of the cut vertex formalism we present the results of calculations of the quantities needed for a next to leading logarithmic analysis of scaling violations for structure and fragmentation functions in Quantum Chromodynamics.We find that an analytic continuation relation holds, between the time and space like regions, while the Gribov-Libatov reciprocity relation is violated.

In this letter the results of calculations needed of the hard scattering processes (a) $e^- h \to e^- + X$ and (b) $e^+e^- \to \bar{h} + X$ are given, in the framework of the cut vertex formalism[1], including next to leading logarithmic corrections in perturbative Q.C.D.

The next to the leading logarithmic approximation is necessary in order to fix the scales in terms of which both the logarithmic and the power corrections are expressed.

Let $F(x,Q^2)$ and $\bar{F}(x,Q^2)$ represent non singlet deep inelastic and annihilation structure function respectively (for example $\nu W_2^{e\,p\to e\,x} - \nu W_2^{e\,n\to e\,x}$ for F and $\nu\bar{W}_2^{e^+e^-\to\pi^+x} - \nu\bar{W}_2^{e^+e^-\to\pi^0x}$ for $\bar{F}$). Then the Q^2 evolution of the moments is given by the relations[1],

$$M_\sigma(Q^2) \equiv \int_0^1 dx\, x^{\sigma-1} F(x,Q^2) = A_\sigma \cdot C_\sigma(1,\bar{\alpha}(t))\, e^{-\int_0^t d\xi\, \gamma_\sigma(\alpha(\xi))} + O\left(\frac{1}{Q^2}\right) \qquad (1)$$

and

$$\bar{M}_\sigma(Q^2) \equiv \int_1^\infty dx\, x^{-\sigma-1}\, \bar{F}(x,Q^2) = \bar{A}_\sigma\, \bar{C}_\sigma(1,\bar{\alpha}(t))\, e^{-\int_0^t \bar{\gamma}_\sigma(\bar{\alpha}(\xi))d\xi} + O\left(\frac{1}{Q^2}\right), \quad (2)$$

where $t = \log \frac{Q^2}{\mu^2}$, $\alpha_s(t)$ is the QCD running coupling constant, C_σ , $\bar{C}_\sigma$, γ_σ, $\bar{\gamma}_\sigma$, the non-singlet coefficient functions and anomalous dimensions for the space like and time like cut vertices. These quantities are calculable in perturbation theory in contrast with the quantities A_σ and $\bar{A}_\sigma$ which must be determined by experiment[2].

Following ref. [3] we define the renormalization group invariant quantity $\bar{K}(x,\alpha)$:

$$\bar{K}_\sigma(\alpha) \equiv \int_1^\infty dx\, x^{-\sigma-1}\, \bar{K}(x,\alpha) \equiv -\bar{\gamma}_\sigma(\alpha) + \beta(\alpha)\left(\frac{\partial}{\partial\alpha}\bar{C}_\sigma(1,\alpha)\right)\Big/\bar{C}_\sigma(1,\alpha). \quad (3)$$

These quantities control the scaling violations of ratios of moments,

$$\bar{R}_\sigma(Q^2,Q_0^2) \equiv \frac{\bar{M}_\sigma(Q^2)}{\bar{M}_\sigma(Q_0^2)} = \exp\left[-\int_{t_0}^t d\xi\, \bar{K}_\sigma(\bar{\alpha}(\xi))\right]. \quad (4)$$

They have also a physical meaning as fractional variations of the structure functions with Q^2 and x [3]:

$$Q^2 \frac{\partial}{\partial Q^2}\bar{F}(x,Q^2) = \int_1^x \frac{dy}{y}\, \bar{K}\left(\frac{x}{y},\bar{\alpha}\right)\, \bar{F}(y,Q^2). \quad (5)$$

Our results will be presented in the inverse Mellin transform space so we introduce the quantities $\bar{e}(x,\alpha)$ and $\bar{P}(x,\alpha)$:

$$\int_1^\infty dx\, x^{-\sigma-1}\, \bar{e}(x,\alpha) = \bar{C}_\sigma(1,\alpha)\,, \quad (6.a)$$

$$\int_1^\infty dx\, x^{-\sigma-1}\, \bar{P}(x,\alpha) = -\bar{\gamma}_\sigma(\alpha)\,. \quad (6.b)$$

The quantity $\bar{e}(x,\alpha)$ has the interpretation of hard scattring cross section and it is process dependent, while $\bar{P}(x,\alpha)$ is process independent.

In the above notation the expansion of $\bar{K}(x,\alpha)$ up to the order α^2 is the following :

$$\bar{K}(x,\alpha) = \frac{\alpha}{4\pi}\,\bar{P}_0(x) + \left(\frac{\alpha}{4\pi}\right)^2\left(\bar{P}_1(x) + b_0\,\bar{e}_1(x)\right) + \cdots , \qquad (7)$$

where

$$\bar{P}(x,\alpha) = \frac{\alpha}{4\pi}\,\bar{P}_0(x) + \left(\frac{\alpha}{4\pi}\right)^2\bar{P}_1(x) + \cdots \qquad (8)$$

$$\bar{e}(x,\alpha) = \bar{e}_0(x) + \frac{\alpha}{4\pi}\,\bar{e}_1(x) + \cdots \qquad (9)$$

and b_0 is the first non zero coefficient of the β-function,

$$\beta(g)/g = \frac{\alpha}{4\pi}\,b_0 + \left(\frac{\alpha}{4\pi}\right)^2 b_1 , \qquad (10a)$$

which is equal to,

$$b_0 = -\frac{1}{3}(11\,C_A - 4\,T_R) , \quad b_1 = -\frac{34}{3}C_A^2 + \frac{20}{3}C_A T_R + 4C_F T_R . \qquad (10b)$$

In the space like case we have relations analogous to the equations (3) to (9) for the corresponding quantities without the symbol bar (-).

We have recalculated the coefficient function $e_1^{(L)}(x)$, $e_1^{(2)}(x)$ for deep inelastic[4,5] and $\bar{e}_1^{(L)}(x)$, $\bar{e}_1^{(2)}(x)$ for annihilation[6] and our results agree with the results of the authors of refs.[4,5,6].We calculated for space like and time-like cut vertices the quantities $P_1(x)$ and $\bar{P}_1(x)$ in the minimal subtraction scheme and in the Feynman gauge. We find that :

i) The moments of $P_1(x)$ agree diagram by diagram after summation over all possible intermediate states with the results of ref.[7]. The check has been done using the simple analytical expression for the anomalous dimensions of Wilson operators [8]. This equivalence of space like cut vertices and minimal twist Wilson operators is expected to be true to all orders[3].

ii) The infrared singularities of soft gluon intermediate states are regularized dimensionally after u.v. divergences have been subtracted and they are found to be cancelled diagram by diagram taking into accound all the possible cuts.

iii) The quantity $\bar{P}_1(x)$ have been extracted by looking how each particular integral is modified in the time like region. We have found the following interesting result which is a kind of analytic continuation[9]. If we represent

the contribution of a diagram G to $P_1(x)$ and $\bar{P}_1(x)$ as

$$P_{1,G}(x) = A_G \delta(1-x) + B_G(x) \quad , \quad 0 \leq x \leq 1 \tag{11.a}$$

and

$$\bar{P}_{1,G}(x) = \bar{A}_G \delta(1-x) + \bar{B}_G(x) \quad 1 \leq x \leq \infty \tag{11.b}$$

then

$$A_G = \bar{A}_G \tag{11.c}$$

and

$$\text{Real Part}\,[B_G(x) + \bar{B}_G(x)] = 0 \quad , \quad 0 \leq x < 1 \tag{11.d}$$

The same relation is true for the one loop coefficient functions $e_1(x)$ and $\bar{e}_1(x)$.

For distributions which appear in $P_{1,G}(x)$ as for example $\left(\frac{x \log(1-x)}{1-x}\right)_+$, equations (11) are easily verified at the level of the infrared regularization using relations like

$$\frac{1}{(x-1)^{1-\varepsilon}} = \frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(x-1)_+} + \varepsilon\left(\frac{\log(x-1)}{x-1}\right)_+ + \cdots, \quad 1 < x < \infty \tag{12.a}$$

and for $0 \leq x < 1$,

$$\text{Re}\left[-\frac{1}{(x-1)^{1-\varepsilon}}\right] = \frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left[\left(\frac{\log(1-x)}{1-x}\right)_+ - \frac{\pi^2}{2}\delta(1-x)\right] . \tag{12.b}$$

iv) The Gribov-Lipatov reciprocity relation[10] is violated for both $e_1(x)$, $\bar{e}_1(x)$ and $P_1(x)$, $\bar{P}_1(x)$ as well as for the renormalization group invariant quantities $K(x,\alpha)$ and $\bar{K}(x,\alpha)$.

Our results for the contributions to $P_1(x)$, $\bar{P}_1(x)$ of the different diagrams can be expressed in terms of logarithms and the following functions[14],

$$S(x) \equiv \int_x^1 \frac{d\xi}{\xi} \log(1-\xi) \equiv Li_2(x) - Li_2(1) \tag{13.a}$$

$$F(x) \equiv \int_{\frac{x}{1+x}}^{\frac{1}{1+x}} \frac{d\xi}{\xi} \log\frac{(1-\xi)}{\xi} = \frac{1}{2}\log^2 x - 2\log x \log(1+x) - 2Li_2(-x) - Li_2(1) , \tag{13.b}$$

and of the "plus" distributions defined by their actions on the test function $\varphi(x)$ as [11] :

$$\int_0^1 dx\, [f(1-x)]_+ \varphi(x) \equiv \int_0^1 dx\, f(1-x)\,[\varphi(x) - \varphi(1)] \tag{14.a}$$

and

$$\int_1^\infty dx\, [f(x-1)]_+ \varphi(x) \equiv \int_1^\infty dx\, f(x-1)\,[\varphi(x) - \theta(2-x)\,\varphi(1)] \tag{14.b}$$

The general expression of a given diagram G is written as

$$\varepsilon\,[\varepsilon\cdot A\,\delta(1-x) + B(x) + \vartheta\cdot C(x)]\,, \tag{15}$$

where $\varepsilon = 1, \vartheta = 0$ for the space like case, while $\varepsilon = -1, \vartheta = -1$ for the time like case.

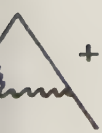
We list here diagram by diagram, the contribution to $P_1(x)$ and $\bar{P}_1(x)$:

$$8 C_F T_R \cdot \varepsilon \left[-\tfrac{4}{9}\varepsilon\,\delta(1-x) + \tfrac{22}{9}(1-x) + \tfrac{2}{3}(1-x)\log x \right] \tag{D.1}$$

$$+ \text{C.C.} \qquad 4 C_F C_A \cdot \varepsilon \left[\tfrac{8}{9}\varepsilon\,\delta(1-x) - \tfrac{61}{9}(1-x) - \tfrac{5}{3}(1-x)\log x \right]. \tag{D.2}$$

$$+ \text{C.C.} \qquad 8\left(C_F^2 - \tfrac{1}{2} C_F C_A\right) \cdot \varepsilon \left[10(1-x) + (7-2x)\log x + (2-x)\,\mathrm{Re}\log(1-x) + \log^2 x \right]. \tag{D.3}$$

$$+ \text{C.C.} \qquad 8 C_F^2 \cdot \varepsilon \left[2(1-x) + (1-x)\,\mathrm{Re}\log(1-x) \right]. \tag{D.4}$$



$$+ \text{C.C.} \qquad 4 C_F C_A \cdot \varepsilon \left[-2(1-x) + (1+3x)\log x + 2(1-x)\,\mathrm{Re}\log(1-x) + x\log^2 x + (1-x)\log^2|1-x| - \vartheta\cdot(1-x)\,\pi^2 \right]. \tag{D.5}$$

$$8\,C_F^2\,.\,\varepsilon\left[-3(1-x)-2\log x-\frac{1}{2}(1+x)\log^2 x\right]$$

(D.

$$8\left(C_F^2-\frac{1}{2}C_F C_A\right).\varepsilon\left[2(1-x)+2x\log x+x\log^2 x\right]$$

(D.

$$8\left(C_F^2-\frac{1}{2}C_F C_A\right).\varepsilon\left[-4(1-x)-4\log x-(1+x)\log^2 x\right]$$

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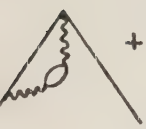
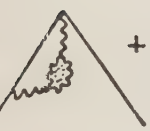


+ c.c.

$$8\,C_F T_R\left[\frac{80}{9}\left(\frac{\varepsilon x}{1-x}\right)_+ + \frac{4}{3}\left(\frac{\varepsilon x\log x}{1-x}\right)_+\right]$$

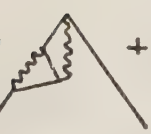
(D.



+ c.c.

$$4\,C_F C_A\left[-\frac{62}{9}\left(\frac{\varepsilon x}{1-x}\right)_+ - \frac{10}{3}\left(\frac{\varepsilon x\log x}{1-x}\right)_+\right]$$

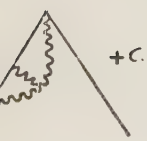
(D.



+ c.c.

$$8\left(C_F^2-\frac{1}{2}C_F C_A\right)\left\{-\left(\frac{\varepsilon x}{1-x}\right)_+ + 3\left(\frac{\varepsilon x\log x}{1-x}\right)_+ + \left(\frac{\varepsilon x\log^2 x}{1-x}\right)_+ - \left(\frac{\varepsilon x\log|1-x|}{1-x}\right)_+ + \varepsilon\left(3\,\mathrm{Re}\log(1-x)\right)\right.$$
$$\left.+\,\varepsilon\left(-\varepsilon\,.\,3\delta(1-x)-\vartheta\,\frac{\pi^2}{2}\,\delta(1-x)\right)\right\}$$

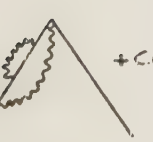
(D.



+ c.c.

$$4\,C_F C_A\,.\left\{-3\left(\frac{\varepsilon x}{1-x}\right)_+ + \left(\frac{\varepsilon x\log x}{1-x}\right)_+ + \frac{1}{2}\left(\frac{\varepsilon x\log^2|1-x|}{1-x}\right)_+ - \vartheta\,.\,\frac{\pi^2}{2}\left(\frac{\varepsilon x}{1-x}\right)_+ + \varepsilon\left[\varepsilon\delta(1-x)-\frac{1}{2}\log^2|1-x|\right.\right.$$
$$\left.\left.+\,\vartheta\,\frac{\pi^2}{2}\right]\right\}$$

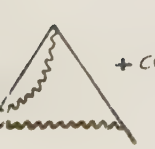
(D.



+ c.c.

$$8\,C_F^2\left[\left(\frac{\varepsilon x}{1-x}\right)_+ + \left(\frac{\varepsilon x\log|1-x|}{1-x}\right)_+ + \vartheta\,.\,\varepsilon\,.\,\frac{\pi^2}{2}\,\delta(1-x)\right]$$

(D.

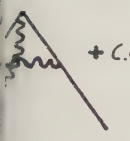


+ cc

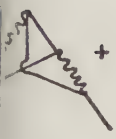
$$8\,C_F^2\,.\,\varepsilon\left[-6(1-x)-2x\log x-4(1-x)\,\mathrm{Re}\log(1-x)+x\log^2 x-(1-x)\log^2|1-x|+(1-x)\,.\,\vartheta\,.\,\pi^2\right.$$
$$\left.-\,4(1-x)\,S(1-x)\right]$$

(D

$$4C_F C_A \cdot \Big\{ \varepsilon\Big[-\varepsilon\delta(1-x) + x\log x - (1+x)\,\mathrm{Re}\log(1-x) - 2(1-x)\big(\mathrm{Re}\,S(x) - S(1-x)\big) - \tfrac{1}{2}\log^2|1-x| + \vartheta\tfrac{\pi^2}{2}\Big] - \tfrac{1}{2}\left(\frac{\varepsilon x\log^2 x}{1-x}\right)_+ - 2\left(\frac{\varepsilon x\,\mathrm{Re}\,S(x)}{1-x}\right)_+ \Big\} \qquad \text{(D.15}$$



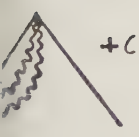
+ C.C. $$8\left(C_F^2 - \tfrac{1}{2}C_F C_A\right)\Big\{\varepsilon\Big[-\varepsilon\Big(6 - \tfrac{2\pi^2}{3} + 2\mathrm{Li}_3(1)\Big)\delta(1-x) - \tfrac{\pi^2}{3}(1-x) - 2(1-2x)\log x - 2x\,\mathrm{Re}\log(1-x) - x\log^2 x + (1-x)\log^2|1-x| - (1-x)\vartheta\pi^2 - 2(1-x)\big(\mathrm{Re}\,S(x) - S(1-x)\big)\Big] - 4\tfrac{\pi^2}{6}\left(\frac{\varepsilon x}{1-x}\right)_+ + \left(\frac{\varepsilon x\log^2 x}{1-x}\right)_+ - 4\left(\frac{\varepsilon x S(1-x)}{1-x}\right)_+\Big\} \qquad \text{(D.16)}$$



+ C.C. $$8\left(C_F^2 - \tfrac{1}{2}C_F C_A\right)\varepsilon\Big[+2(1-x)\log x + (1+x)\log^2 x - 2(1+x)F(x) + \frac{4x}{1+x}F(x)\Big] \qquad \text{(D.17)}$$

$$0 \qquad \text{(D.18}$$

+ C.C. $$4C_F C_A\Big\{\varepsilon\Big[2\,\mathrm{Re}\log(1-x) + \log^2|1-x| - \vartheta\pi^2\Big] - 2\left(\frac{\varepsilon x}{1-x}\right)_+ - 2\left(\frac{\varepsilon x\log x}{1-x}\right)_+ - \tfrac{1}{2}\left(\frac{\varepsilon x\log^2 x}{1-x}\right)_+ - 2\left(\frac{\varepsilon x\cdot\mathrm{Re}\,S(x)}{1-x}\right)_+ + \left(\frac{\varepsilon x\log|1-x|}{1-x}\right)_+ + \tfrac{1}{2}\left(\frac{\varepsilon x\log^2|1-x|}{1-x}\right)_+ + \vartheta\Big(\varepsilon\tfrac{\pi^2}{2}\delta(1-x) - \tfrac{\pi^2}{2}\left(\frac{\varepsilon x}{1-x}\right)_+\Big)\Big\} \qquad \text{(D.1}$$



+ CC $$8\left(C_F^2 - \tfrac{1}{2}C_F C_A\right)\Big\{\varepsilon\Big[\varepsilon 4\delta(1-x) + 4\,\mathrm{Re}\log(1-x)\Big] - 2\left(\frac{\varepsilon x\log^2 x}{1-x}\right)_+ - 4\left(\frac{\varepsilon x\,\mathrm{Re}\,S(x)}{1-x}\right)_+\Big\} - 4C_F C_A\Big[4\left(\frac{\varepsilon x}{1-x}\right)_+ + 4\left(\frac{\varepsilon x S(1-x)}{1-x}\right)_+ + \left(\frac{\varepsilon x\log^2 x}{1-x}\right)_+ + 2\left(\frac{\varepsilon x\log|1-x|}{1-x}\right)_+ + \left(\frac{\varepsilon x\log^2|1-x|}{1-x}\right)_+ + \vartheta\Big(\varepsilon\pi^2\delta(1-x) - \pi^2\left(\frac{x\varepsilon}{1-x}\right)_+\Big)\Big] \qquad \text{(D.2}$$

+ $\frac{1}{2}$ CC $$\Big[\frac{31}{2}C_F C_A - 3\left(C_F^2 - \tfrac{1}{2}C_F C_A\right) - 4C_F T_R\Big]\cdot\varepsilon\cdot\big[\varepsilon\delta(1-x)\big] \qquad \text{(D.2}$$

The results for P and $\bar{P}$ verify, as mentioned, the continuation relations 11.

Before we present the total result we should also mention the following. It is known that in the higher loops calculation (here two loops) there is a mixing of quark and antiquark densities[12]. Looking at the diagrams D7 and D8 (or D16 and D17) we see that they contribute to discontinuities in different channels of the same diagram. So they give contributions to the probability to find quark inside a quark ($Q_{qq}(x)$) or antiquark inside a quark ($Q_{q\bar{q}}(x)$). Thus we must separate these contributions also for $P(x,\alpha)$ and $\bar{P}(x,\alpha)$. We call them $P_{qq}(x,\alpha)$ and $P_{q\bar{q}}(x,\alpha)$ and similarly for $\bar{P}(x,\alpha)$. As it is explained in ref. [12], when we study scaling violations of structure functions of definite parity $\eta = \pm 1$, under crossing $x \to -x$, we have to consider in the kernel of equation (5) the combinations,

$$\bar{P}(x,\alpha) = P_{qq}(x,\alpha) + \eta\, P_{q\bar{q}}(x,\alpha) \,. \tag{16}$$

This is the meaning of η appearing in the total result. Quite recently another group calculated $e_1(x)$, $\bar{e}_1(x)$, $P_1(x)$, $\bar{P}_1(x)$ in a different calculational framework[13]. In particular they used a light-like axial gauge and different normalization of the coefficient functions in the modified minimal substraction scheme. Thus the comparison with our results cannot be done neither diagram by diagram, nor for the total answer $\bar{P}_1(x)$ or $\bar{e}_1(x)$. We did the comparison for the renormalization group invariant function $\bar{K}(x,\bar{\alpha})$ and we find agreement. The sum of all the contributions for $P_1(x)$ and $\bar{P}_1(x)$ is :

$$\left.\begin{matrix} P_1(x) \\ \bar{P}_1(x) \end{matrix}\right\} = -8C_F \left\{ \frac{2}{3} T_R \cdot \varepsilon \left[2(1-x) + P_0(x) \left(\frac{5}{3} + \log x \right) \right] + \right.$$

$$+ C_F \cdot \varepsilon \left[5(1-x) + \frac{1}{2}(3+7x)\log x + \frac{1}{2}(1+x)\log^2 x + \right.$$

$$\left. + P_0(x) \log x \left(\frac{3}{2} + 2\log|1-x| \right) \right] +$$

$$+ \frac{1}{2} C_A \cdot \varepsilon \left[-\frac{40}{3}(1-x) - 2(1+x)\log x - P_0(x) \left(\log^2 x + \frac{11}{3}\log x + \right. \right.$$

$$\left. \left. + \frac{67}{9} - \frac{\pi^2}{3} \right) \right] - \eta \left(C_F - \frac{1}{2} C_A \right) \cdot \varepsilon \cdot \left[4(1-x) + 2(1+x)\log x + 2P_0(-x)F(x) \right] \Bigg\} \tag{17}$$

+

where $P_o(x) = \frac{1+x^2}{1-x}$ and $\varepsilon = 1$ for the $P_1(x)$ and $\varepsilon = -1$ for the $\bar{P}_1(x)$.

The results for the coefficient functions $e^{(2)}(x)$, $e^{(L)}(x)$, $\bar{e}^{(2)}(x)$ and $\bar{e}^{(L)}(x)$ are :

$$\left.\begin{matrix} e_o^{(L)}(x) \\ \bar{e}_o^{(L)}(x) \end{matrix}\right\} = 0 \quad , \quad \left.\begin{matrix} e_o^{(2)}(x) \\ \bar{e}_o^{(2)}(x) \end{matrix}\right\} = \delta(1-x) \quad , \quad \left.\begin{matrix} e_1^{(L)}(x) \\ \bar{e}_1^{(L)}(x) \end{matrix}\right\} = C_F \cdot \varepsilon \cdot 2x \qquad (18)$$

$$\left.\begin{matrix} e^{(2)}(x) \\ \bar{e}^{(2)}(x) \end{matrix}\right\} = C_F \left\{ \left\{ \varepsilon \left[\tfrac{1}{2}(5x+9) + P_o(x)\left(2\log\frac{|1-x|}{x} - \tfrac{3}{2}\right)\right]\right\}_+ - \vartheta \cdot 2\pi^2 \delta(1-x) \right\}, \qquad (19)$$

where $\varepsilon = 1$ and $\vartheta = 0$ for $e(x)$ and $\varepsilon = -1, \vartheta = -1$ for $\bar{e}(x)$.

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- VII -

CORRECTIONS D'ORDRES SUPERIEURS

POUR LES TRANSITIONS SINGULET

- 1. La relation de continuation analytique

Dans le cas non-singulet, que nous avons traité au chapitre précédent, nous avons trouvé à l'ordre $\alpha_s^2(Q)$, une relation analytique entre les noyaux de diffusion et d'annihilation.

On peut démontrer que cette relation reste valable, dans le cas singulet. Elle s'applique aussi bien pour les fonctions coefficients que pour les éléments de matrice des noyaux d'évolution $\tilde{P}_{ij}(x,\alpha)$. Une fonction de structure singulet satisfait à l'équation suivante:

$$F^s(x,Q) = E_q^F(x,\alpha(Q)) \otimes \Sigma(x,Q) + E_G^F(x,\alpha(Q)) \otimes G(x,Q) \qquad (1)$$

où $\Sigma(x,Q)$ est la somme de toutes les distributions des quarks et antiquarks, et $G(x,Q)$ est la distribution des gluons.

$$\begin{pmatrix} \dot{\Sigma} \\ \lambda \dot{G} \end{pmatrix} = \begin{pmatrix} P_{qq}^s & P_{qG} \\ P_{Gq} & P_{GG} \end{pmatrix} \otimes \begin{pmatrix} \Sigma \\ \lambda G \end{pmatrix}, \quad \lambda = 1 \qquad (2)$$

On peut obtenir des équations semblables à (1) et (2) pour $\tilde{F}^s(x,Q)$, $\tilde{E}_q^F(x,\alpha)$, $\tilde{P}_{ij}(x,\alpha)$, avec un $\tilde{\lambda}$ pris égal à $-\frac{C_F}{2T_R}$ pour tenir compte de la moyenne sur les états de polarisation des gluons et des quarks.

Nous avons trouvé les relations de continuation analytique suivantes:

$$P_{ij}(x,\alpha) = -\left[\text{Partie Réelle} \quad \tilde{P}_{ij}(x,\alpha) \; ; \; x<1\right] \qquad (3a)$$

et

$$E_i^F(x,\alpha) = -\left[\text{Partie Réelle} \quad \tilde{E}_i^F(x,\alpha) \; ; \; x<1\right] \qquad (3b)$$

Dans l'espace des moments, celles-ci s'écrivent:

$$\langle A \rangle_\sigma = \text{Partie Réelle}\left[\langle \tilde{A} \rangle_\sigma \; ; \; \sigma \to -\sigma\right] \qquad (4)$$

où $A(x)$ est relatif à la diffusion $\left|\begin{array}{l} x<1 \\ Re\ \sigma>0 \end{array}\right.$ (5a)

et $\tilde{A}(x)$ à l'annihilation $\left|\begin{array}{l} x>1 \\ Re\ \sigma>0 \end{array}\right.$ (5b)

La dernière relation nous permet d'évaluer les $E_i^F(x,\alpha)$, $P_{ij}(x,\alpha)$, $\tilde{E}_i^F(x,\alpha)$ et $\tilde{P}_{ij}(x,\alpha)$, à partir des dimensions anormales $\gamma_{ij}^{\sigma}(\alpha)$ et des fonctions coefficients $C_{Fi}^{\sigma}(\alpha)$ des opérateurs de twist minimum du développement sur le cône de lumière.

On peut, en effet, trouver $P_{ij}(x,\alpha)$ à partir de $\gamma_{ij}^{\sigma}(\alpha)$ en faisant la transformation de Mellin inverse, puisqu'on a démontré au chapitre (V) que les opérateurs de twist minimum sont identiques aux vertex coupés de genre espace:

$$-\gamma_{ij}^{\sigma}(\alpha) = \langle P_{ij} \rangle_{\sigma} \xrightarrow{T.M.I} P_{ij}(x,\alpha) \tag{6}$$

D'autre part, à partir de $\langle P_{ij} \rangle_{\sigma}$ on peut trouver $\langle \tilde{P}_{ij} \rangle_{\sigma}$ en utilisant la relation de continuation analytique (4). $\tilde{P}_{ij}(x,\alpha)$ s'obtient alors par une transformation de Mellin inverse ($x>1$) :

$$\langle P_{ij} \rangle_{\sigma} \xrightarrow{Cont.\ Anal.} \langle \tilde{P}_{ij} \rangle_{\sigma} \xrightarrow{T.M.I.} \tilde{P}_{ij}(x,\alpha) \tag{7}$$

Toutes les formules nécessaires, relatives à la continuation analytique et à la transformée inverse de Mellin, se trouvent dans l'Appendice (B).

SPACE AND TIME LIKE CUT VERTICES IN QCD BEYOND THE LEADING ORDER (II) THE SINGLET SECTOR

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ABSTRACT

The analytic continuation relation found in the non-singlet structure function of the processes $e^+h \to e^-X$ and $e^-e^+ \to hX$ up to the next to the leading logarithmic approximation, is shown to be true also for the singlet case. This relation is used to find the two loop anomalous dimensions for the singlet time like cut vertices from the anomalous dimension of the singlet space like cut vertices. We give also the singlet time like coefficient functions.

Recently the applicability region of perturbative QCD has been extended to almost all hard scattering (semi) inclusive [1,2,3] and exclusive processes [4,5]. Renormalization group methods can be applied, in almost all cases, to sum the logarithms of large invariants to all orders of perturbation expansion. Already the next to the leading logarithms have been calculated in the time like region for the flavour non-singlet channel of the process $e^+e^- \to h + X$ [6,7].

In a previous work [7] we found, up to two loops, a simple analytic continuation relation between the inverse Mellin transform of anomalous dimensions for the processes $e^-h \to e^-X$ and $e^-e^+ \to \bar{h}X$. Let $F_{NS}(X,q^2)$, $\bar{F}_{NS}(X,q^2)$ non-singlet structure functions for the above processes correspondingly. Then the evolution of these functions with q^2 is given by the equations [8] :

$$q^2\frac{\partial}{\partial q^2}F_{NS}(x,q^2) = \left[e^{-1}_{NS}(x,\bar{\alpha}) \underset{S.L.}{\otimes} q^2\frac{\partial}{\partial q^2}e_{NS}(x,\bar{\alpha}) + P_{NS}(x,\bar{\alpha})\right] \underset{S.L}{\otimes} F_{NS}(x,q^2), \tag{1a}$$

$$q^2\frac{\partial}{\partial q^2}\bar{F}_{NS}(x,q^2) = \left[\bar{e}^{-1}_{NS}(x,\bar{\alpha}) \underset{T.L.}{\otimes} q^2\frac{\partial}{\partial q^2}\bar{e}_{NS}(x,\bar{\alpha}) + \bar{P}_{NS}(x,\bar{\alpha})\right] \underset{T.L.}{\otimes} \bar{F}_{NS}(x,q^2), \tag{1b}$$

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where $\bar{\alpha} = \bar{\alpha}(|q^2|)$ the running coupling constant, the convolution $\otimes$ mean

$$g_1(x) \underset{SL}{\otimes} g_2(x) = \int_x^1 \frac{dy}{y} g_1(y) g_2\left(\frac{x}{y}\right), \quad g_1(x) \underset{T.L.}{\otimes} g_2(x) = \int_1^x \frac{dy}{y} g_1(y) g_2\left(\frac{x}{y}\right) \quad (2)$$

and

$$\int_0^1 dx\, x^{\sigma-1} e_{NS}(x,\bar{\alpha}) = C_{NS}^{\sigma}(1,\bar{\alpha}), \quad \int_1^\infty dx\, x^{-\sigma-1} \bar{e}_{NS}(x,\bar{\alpha}) = \bar{C}_{NS}^{\sigma}(1,\bar{\alpha}); \quad (3a)$$

$$\int_0^1 dx\, x^{\sigma-1} P_{NS}(x,\bar{\alpha}) = -\gamma_{NS}^{\sigma}(\bar{\alpha}), \quad \int_1^\infty dx\, x^{-\sigma-1} \bar{P}_{NS}(x,\bar{\alpha}) = -\bar{\gamma}_{NS}^{\sigma}(\bar{\alpha}), \quad (3b)$$

with C_{NS}^{σ}, C_{NS}^{σ} the non-singlet coefficient functions and γ_{NS}^{σ}, γ_{NS}^{σ} the non-singlet anomalous dimensions.

What we call analytic continuation relation [9] is the following: the kernels $P(x,\bar{\alpha})$ and $\bar{P}(x,\bar{\alpha})$ have the following forms:

$$P(x,\bar{\alpha}) = P_A\, \delta(1-x) + P_B(x); \quad 0 \leq x \leq 1 \quad (4a)$$

$$\bar{P}(x,\bar{\alpha}) = \bar{P}_A\, \delta(1-x) + \bar{P}_B(x); \quad 1 \leq x < \infty \quad (4b)$$

What we find is that

$$P_A = \bar{P}_A \quad (5a)$$

and

$$P_B(x) = -Re\, \bar{P}_B(x); \quad 0 < x < 1 \quad (5b)$$

Similar relation are valid for $\bar{e}_{NS}(x)$ and $e_{NS}(x)$ [8]. The relations (5a,5b) have been obtained in the non-singlet case doing the calculations in the Feynman gauge and in the minimal substraction scheme for both the space like and time like cut vertices. The kernels $P_{NS}(x,\bar{\alpha})$ and $\bar{P}_{NS}(x,\bar{\alpha})$ are the coefficients of the simple pole parts of the regularized cut vertices after substraction of the corresponding lower order counter terms.

The particular cuts of a diagram may have infrared divergences coming from the production of soft massless gluons. These divergences ($x \to 1$) are regularized dimensionally and they are found to cancel diagram by diagram after

summation over all possible cuts. In the space like region we found that the contribution of each particular diagram to $P_{NS}(x,\alpha)$ is identical to that obtained from the inverse Mellin transform of anomalous dimensions of minimal twist Wilson operators [10]. In fact, if we decompose the Feynamn integral of every diagram into a set of scalar integrals, then, the equality between space like cut vertices and Wilson operators is still valid. This result is to be expected for reasons of analytic properties of Feynman integrals [8], and can be easily seen from the following formulae :

i) Space like cut vertex generic integral.

$$\left\{A,B\right\}_{SL} \equiv \int \frac{d^\eta k}{(2\pi)^\eta} \frac{\delta(x-\frac{\Delta k}{\Delta p})\,\theta(\frac{\Delta k}{\Delta p})\,f(\frac{\Delta k}{\Delta p})}{(-k^2-i\varepsilon)^A} \cdot abs\,\frac{1}{(-(p-k)^2-i\varepsilon)^B}$$

$$= \frac{i}{(4\pi)^{\eta/2}} \cdot \frac{\Gamma(A+B-\frac{\eta}{2})}{\Gamma(A)\,\Gamma(B)} \cdot (-P^2+i\varepsilon)^{\frac{\eta}{2}-A-B} \cdot f(x)\, x^{\frac{\eta}{2}-A-1}(1-x)^{\frac{\eta}{2}-B-1}, \qquad (6a)$$

where $P^2 < 0$, $0 < x < 1$ and $\Delta^2 = 0$.

ii) Minimal twist Wilson operator generic integral.

$$\left\{A,B\right\}_{W} \equiv \int \frac{d^\eta k}{(2\pi)^\eta} \frac{\delta(x-\frac{\Delta k}{\Delta p})\,f(\frac{\Delta k}{\Delta p})}{(-k^2-i\varepsilon)^A\,(-(p-k)^2-i\varepsilon)^B}$$

$$= \frac{i}{(4\pi)^{\eta/2}} \cdot \frac{\Gamma(A+B-\frac{\eta}{2})}{\Gamma(A)\cdot\Gamma(B)} \cdot (-P^2+i\varepsilon)^{\frac{\eta}{2}-A-B} \cdot \int_0^1 da\,\delta(x-a)\,f(a)\,a^{\frac{\eta}{2}-A-1}(1-a)^{\frac{\eta}{2}-B-1} \qquad (6b)$$

where a is a Feynman parameter and $P^2 < 0$, $\Delta^2 = 0$.

In the same way the analytic continuation relation 5a, 5b holds for each of these scalar integrals, (after the inclusion of ultra-violet counter terms and summation of all possible cuts), as a consequence of the result for the generic integral in the time like region.

iii) Time like cut vertex generic integral :

$$\left\{A,B\right\}_{TL} \equiv \int \frac{d^\eta k}{(2\pi)^\eta} \frac{\delta(x-\frac{\Delta k}{\Delta p})\,f(\frac{\Delta k}{\Delta p})}{(-k^2-i\varepsilon)^A}\; abs\,\frac{1}{(-(p-k)^2-i\varepsilon)^B}$$

$$= \frac{i}{(4\pi)^{\eta/2}}\, e^{i\pi A}\, \frac{\Gamma(A+B-\frac{\eta}{2})}{\Gamma(A)\cdot\Gamma(B)}\, (p^2)^{\frac{\eta}{2}-A-B} \cdot f(x)\, x^{\frac{\eta}{2}-A-1}\,|1-x|^{\frac{\eta}{2}-B-1}, \qquad (7a)$$

or equivalently

$$\{A,B\}_{TL} = -\frac{i}{(4\pi)^{n/2}} \cdot \frac{\Gamma(A+B-\frac{n}{2})}{\Gamma(A)\,\Gamma(B)} \cdot (-P^2+i0)^{\frac{n}{2}-A-B} \int f(x)\, x^{\frac{n}{2}-A-1} (1-x-i0)^{\frac{n}{2}-B-1} \, , \tag{7b}$$

where $P^2 > 0$, $\infty < x < 1$ and $\Delta^2 = 0$.

We observe that $\{A,B\}_{SL}$ and $-\{A,B\}_{TL}$ are related by analytic continuation in the variables x and P^2. In the non-singlet case we have been able to obtain all two loops integrals by successive application of the generic integrals (6a, 6b and 7b) over internal variables K_i. After the K_i-loop integrations we are left with convolution integrals in the variables $x_i = \frac{\Delta \cdot K_i}{\Delta \cdot P}$. These convolutions are not trivial operations but they have the property of conserving the relationship between 6a and 7b. This relationship (analytic continuation in the variables x and P^2) is the origin of analytic continuation (5a, 5b). Indeed for the pole parts, (coefficients of $\frac{1}{(n-4)}$) we are looking for, the factors $(-P^2 + i0)^{-(n-4)}$ cancelled by the inclusion of the counter terms ; the addition of the symmetric diagram (complex conjugate) gives a real contribution.

The sign - of relation (5b) comes from opposite ordering of phase space limits in the time like and space like integrals in relation 6a,7b. Relation (5a) comes from the Feynman rules of cut vertices which correspond to zero particle cuts of diagrams with the rest of the diagram having euclidean momentum integrations. These integrations give the same answer for space like and time like cut-vertices.

Let us note that the Dirac algebra manipulation introduces integrals with loop momentum vectors in the numerator which can be expressed in terms of the generic ones (6-7). For example :

$$\{A,B\}_{SL}(K^\mu) \equiv \int \frac{d^n k}{(2\pi)^n} \frac{\delta(x-\frac{\Delta K}{\Delta P})\,\theta(\frac{\Delta K}{\Delta P})\, f(\frac{\Delta K}{\Delta P}).\, K^\mu}{(-K^2-i0)^A} \cdot abs \frac{1}{(-(P-K)^2-i0)^B}$$

$$= \{A,B\}_{SL} \cdot \times \left(P^\mu - P^2 \frac{\Delta^\mu}{2\Delta P}\right) - \{A-1,B\} \cdot \frac{1}{x}\left(1 + \frac{1-\frac{n}{2}}{A-1}(1-x)\right)\frac{\Delta^\mu}{2\Delta P} . \tag{8}$$

We now come to the application of relations (5a) and (5b).

In ref. 12 the two loop anomalous dimension of the flavour singlet minimal twist Wilson operators have been calculated as well as their coefficient functions. In this work if has been found that no new types of scalar Feynamn integrals were needed in comparison with the flavour non-singlet case. The topology of the Feynman diagrams also is the same as that of the non-singlet case.

These facts imply that the analytic continuation (5a) and (5b) holds for each scalar integral appearing in the singlet cut vertices. Thus by constructing the exact inverse Mellin transform of the singlet anomalous dimensions, we can obtain the inverse Mellin transform of the time like anomalous dimensions from relations (5a) and (5b).

We give also the time-like singlet coefficient functions for completeness All technical details will be given in a future publication.

For the Mellin inversion of the singlet anomalous dimensions we use the simple analytic form of ref. 13 which has been checked to give numerically the results of ref. 12 .

Let $\bar{F}(x,q^2)$ a singlet structure function for the process $e^+e^- \to \bar{h} + X$; then, the factorization property implies [8]

$$\bar{F}(x,q^2) = \int_x^1 \frac{dy}{y} \left[\bar{e}_q\left(\frac{x}{y},\bar{\alpha}\right) D_q(y,q^2) + \bar{e}_g\left(\frac{x}{y},\bar{\alpha}\right) D_g(y,q^2) \right] \qquad (9)$$

where we have defined :

$$\int_x^1 dx\; x^{-\sigma-1}\, \bar{e}_j(x,\bar{\alpha}) = \bar{C}_j^{\,\sigma}(1,\bar{\alpha}) \qquad (10a)$$

$$\int_x^1 dx\; x^{-\sigma-1}\, D_j(x,q^2) \equiv \sum_i T\left[\exp - \int_0^t d\xi\, \bar{\gamma}^{\sigma}(\alpha(\xi))\, d\xi\right]_{ji} \bar{A}_i^{\sigma} \qquad (10b)$$

The i,j stand for the quark or gluon (q,g). $\bar{A}_j$ are the hadronic matrix elements of quark (q) and gluon (g) time like cut vertices.

If we represent by $\bar{F}_q$ and $\bar{F}_g$ respectively the expressions

then from relation 7a-b and 8a-b we find

$$\begin{pmatrix} \dot{\bar{F}}_q \\ \dot{\bar{F}}_g \end{pmatrix} = \begin{pmatrix} \dot{\bar{e}}_q \otimes_{TL} \bar{e}_q^{-1} + \bar{P}_{qq} & \dot{\bar{e}}_q \otimes_{TL} \bar{e}_g^{-1} \otimes_{TL} \bar{P}_{qg} \\ \dot{\bar{e}}_g \otimes_{TL} \bar{e}_q^{-1} \otimes_{TL} \bar{P}_{gq} & \dot{\bar{e}}_g \otimes_{TL} \bar{e}_g^{-1} + P_{gg} \end{pmatrix} \otimes_{TL} \begin{pmatrix} \bar{F}_q \\ \bar{F}_g \end{pmatrix} \quad (11)$$

where $\int_1^\infty dx\, x^{-\sigma-1} \bar{P}_{ij}(x,\alpha) = -\bar{\gamma}_{ij}^{\sigma}(\alpha)$, $i,j = q,g$

the dot in the expression(11) means derivation with respect to $\log\left|\frac{q^2}{\mu^2}\right|$.

The functions $\bar{e}_i(x,\alpha)$ and $\bar{P}_{ij}(x,\alpha)$ have the following expansion in :

$$\bar{e}_j(x,\alpha) = \bar{e}_j^{(0)}(x) + \bar{e}_j^{(1)}(x)\left(\frac{\alpha}{4\pi}\right) + \cdots , \quad (\bar{e}_g^{\,o} = 0) \quad (12a)$$

$$\bar{P}_{ij}(x,\alpha) = \left(\frac{\alpha}{4\pi}\right)\bar{P}_{ij}^{(0)}(x) + \left(\frac{\alpha}{4\pi}\right)^2 \bar{P}_{ij}^{(1)}(x) + \cdots . \quad (12b)$$

Below we give the results for e_i, $\bar{e}_i$, P_{ij} and $\bar{P}_{ij}$.

$$e_{q,L}^{(o)}(x) = \bar{e}_{q,L}^{(o)}(x) = 0 \quad , \quad e_{g,L}^{(o)}(x) = \bar{e}_{g,L}^{(o)}(x) = 0 \quad (13a)$$

$$e_{q,2}^{(o)}(x) = \bar{e}_{q,2}^{(o)}(x) = \delta(1-x) , \quad e_{g,2}^{(o)}(x) = e_{g,2}^{(o)}(x) = 0 \quad (13b)$$

$$\left.\begin{matrix} P_{qq}^{(o)}(x) \\ \bar{P}_{qq}^{(o)}(x) \end{matrix}\right\} = 4C_F\left(\varepsilon R_{qq}(x)\right)_+ , \quad (14a)$$

$$\left.\begin{matrix} P_{qg}^{(o)}(x) \\ \bar{P}_{qg}^{(o)}(x) \end{matrix}\right\} = 8T_R\, \varepsilon R_{qg}(x) , \quad (14b)$$

$$\left.\begin{matrix} P_{gq}^{(o)}(x) \\ \bar{P}_{gq}^{(c)}(x) \end{matrix}\right\} = 4C_F\, \varepsilon R_{gq}(x) , \quad (14c)$$

$$\left.\begin{matrix} P_{gg}^{(o)}(x) \\ \bar{P}_{gg}^{(o)}(x) \end{matrix}\right\} = 4C_A\left[\left(\varepsilon R_{qq}(x)\right)_+ + \varepsilon R_{gq}(x) - \varepsilon R_{qg}(x)\right] + \frac{4}{3}(C_A - 2T_R)\delta(1-x) . \quad (14d)$$

$$\left.\begin{matrix} e^{(1)}_{q,L}(x) \\ \bar{e}^{(1)}_{q,L}(x) \end{matrix}\right\} = 2 C_F \varepsilon x \quad , \quad \left.\begin{matrix} e^{(1)}_{g,L}(x) \\ \bar{e}^{(1)}_{g,L}(x) \end{matrix}\right\} = -8 T_R \varepsilon x(1-x) \quad , \qquad (15a)$$

$$\left.\begin{matrix} e^{(1)}_{q,2}(x) \\ \bar{e}^{(1)}_{q,2}(x) \end{matrix}\right\} = C_F \Big\{ \Big[\varepsilon \tfrac{1}{2}(9+5x) + \varepsilon R_{qq}(x) \cdot \big(2 \log\left|\tfrac{1-x}{x}\right| - \tfrac{3}{2}\big) \Big]_+ - \vartheta \cdot 2\pi^2 \delta(1-x) \Big\} , \qquad (15b)$$

$$\left.\begin{matrix} e^{(1)}_{g,2}(x) \\ \bar{e}^{(1)}_{g,2}(x) \end{matrix}\right\} = -4 T_R \varepsilon \Big[3 + R_{qg} \big(\log\left|\tfrac{1-x}{x}\right| - 4 \big) \Big] .$$

$$\left.\begin{matrix} P^{(1)}_{qq}(x) \\ \bar{P}^{(1)}_{qq}(x) \end{matrix}\right\} = -8 C_F \Big\{ \tfrac{2}{3} T_R \varepsilon \big[2(1-x) + R_{qq}(x) \big(\tfrac{5}{3} + \log x \big) \big] \qquad (15c)$$

$$+ C_F \cdot \varepsilon \big[5(1-x) + \tfrac{1}{2}(3+7x) \log x + \tfrac{1}{2}(1+x) \log^2 x + R_{qq}(x) \log x \big(2 \log|1-x| + \tfrac{3}{2} \big) \big]$$

$$+ \tfrac{1}{2} C_A \cdot \varepsilon \Big[-\tfrac{40}{3}(1-x) - 2(1+x) \log x - R_{qq}(x) \big(\log^2 x + \tfrac{11}{3} \log x + \tfrac{67}{9} - \tfrac{\pi^2}{3} \big) \Big]$$

$$- \eta \big(C_F - \tfrac{1}{2} C_A \big) \varepsilon \big[4(1-x) + 2(1+x) \log x + 2 R_{qq}(-x) F(x) \big] \Big\}_+$$

$$+ \tfrac{16}{3} C_F T_R \varepsilon \Big[\frac{20 - 18x + 54x^2 - 56x^3}{3x} - 3(1+x) \log^2 x + (3 + 15x + 8x^2) \log x \Big] .$$

(16)

$$\left.\begin{matrix} P_{qg}^{(1)}(x) \\ \bar{P}_{qg}^{(1)}(x) \end{matrix}\right\} = -8 T_R \varepsilon \Big\{ C_F \Big[4 - 9x - (1-4x)\log x + 4\log|1-x| + 2x^2 \log^2 x$$
$$+ R_{qg}(x)\left(10 - 4\log\left|\tfrac{1-x}{x}\right| + 2\log^2\left|\tfrac{1-x}{x}\right| - \log^2 x - \tfrac{2\pi^2}{3} - \vartheta 2\pi^2\right)\Big]$$
$$+ C_A \Big[\tfrac{1}{18}\,\frac{80 - 171x + 684x^2 - 557x^3}{x} + \tfrac{1}{3}(6 + 15x + 52x^2)\log x$$
$$- 4\log|1-x| - \tfrac{1}{2}(3+7x)\log^2 x$$
$$+ R_{qg}(x)\left(-2\log^2|1-x| - \tfrac{1}{2}\log^2 x + 4\log|1-x| + \tfrac{\pi^2}{3} + 2\vartheta\pi^2\right)$$
$$+ \eta\Big[\tfrac{1}{2}(1-x)(11+35x) - (6+x)\log x - \tfrac{1}{2}(1+5x)\log^2 x$$
$$+ R_{qg}(-x)\left(6\log x - \tfrac{1}{2}\log^2 x + 2F(x)\right)\Big]\Big]\Big\} \qquad (17)$$

$$\left.\begin{matrix} P_{gq}^{(1)}(x) \\ \bar{P}_{gq}(x) \end{matrix}\right\} = 8 C_F \varepsilon \Big\{ \tfrac{4}{3} T_R \Big[x + R_{gq}\left(\tfrac{5}{3} + \log|1-x|\right)\Big]$$
$$+ \tfrac{1}{2} C_F \Big[5 + 7x - (4+7x)\log x + 4x\log|1-x| + (2-x)\log^2 x$$
$$+ R_{gq}(x)\left(6\log|1-x| + 2\log^2|1-x| - 2\vartheta\pi^2\right)\Big]$$
$$- C_A \Big[\tfrac{1}{18}(38 + 47x + 88x^2) + 2x\log|1-x| + (1+x)\log^2 x$$
$$- \tfrac{2}{3}(15 + 6x + 4x^2)\log x + R_{gq}(x)\left(\tfrac{5}{2} + \tfrac{11}{3}\log|1-x| - 2\log x\log(1-x)\right.$$
$$\left. - \tfrac{\pi^2}{6} + \log^2|1-x| - \vartheta\pi^2\right)$$
$$+ \eta\Big[9 + x + \log^2 x - (2+x)\log x + R_{gq}(-x)\left(F(x) - \tfrac{1}{2}\log^2 x + 2\right)\Big]\Big\} \qquad (18)$$

$$\left.\begin{matrix} P_{gg}^{(1)}(x) \\ \bar{P}_{gg}^{(1)}(x) \end{matrix}\right\} = -\frac{16}{9} C_A T_R \varepsilon \left[5 R_{gg}(x) + 6(1+x)\log x + \frac{13 - 9x + 9x^2 - 13x^3}{x} - 4\varepsilon\delta(1-x) \right]$$

$$- \frac{8}{3} C_F T_R \varepsilon \left[3\varepsilon\delta(1-x) - \frac{4(1-x)}{x}(1 - 11x - 5x^2) + 6(3+5x)\log x \right.$$

$$\left. + 6(1+x)\log^2 x \right]$$

$$- C_A^2 \cdot \varepsilon \left[\varepsilon \frac{56}{9} \delta(1-x) - \frac{513 + 287x + 919x^2 - 783x^3}{18x} \right.$$

$$+ \frac{1}{3}(140 - 118x + 226x^2)\log x - (19 + 39x - 12x^2)\log^2 x$$

$$+ \frac{2(22 - 14x - 13x^2)}{x}\log|1-x| + \frac{2(5 - 5x - 2x^2)}{x}\left(\log^2|1-x| - \vartheta\pi^2\right)$$

$$- 8(1+x)\left(3 \mathrm{Re}\, S(x) + 2S(1-x)\right) - \frac{536}{9}\left(\frac{x}{1-x}\right)_+ - 8\left(\frac{x\log^2 x}{1-x}\right)_+$$

$$+ 8 R_{gg}(x)\left(2\log x \log|1-x| + \frac{\pi^2}{6}\right)$$

$$+ \eta \left[\frac{1-x}{2x}(57 - 16x + 87x^2) + (8 + 3x + 21x^2)\log x \right.$$

$$\left. \left. - \frac{8 + 15x + 17x^2 + 4x^3}{x}\log^2 x - 8 R_{gg}(-x) F(x) \right] \right] \qquad (19)$$

where $\eta = +1$ is the parity under crossing $x \to -x$ of the considered structure functions ; ($\varepsilon = 1$, $\vartheta = 0$) in the space like case ($\varepsilon = -1$, $\vartheta = -1$) in the time like case and

$$R_{qq}(x) = \frac{1 + x^2}{1 - x} \quad , \qquad (20a)$$

$$R_{qg}(x) = x^2 + (1-x)^2 \quad , \qquad (20b)$$

$$R_{gq}(x) = \frac{1 + (1-x)^2}{1 - (1-x)} \quad , \qquad (20c)$$

$R_{gq}(x)$ has to be understood as a distribution defined by its product with a function G(x) as

$$\varepsilon R_{gg}(x)\,G(x) = \left(\frac{\varepsilon\, 2x\,G(x)}{1-x}\right)_+ + \varepsilon\,\frac{2}{x}(1-x+x^2-x^3)\,G(x), \quad (20d)$$

$$S(x) = \int_x^1 \frac{d\xi}{\xi}\log(1-\xi) = Li_2(x) - Li_2(1) \quad (21a)$$

$$F(x) = \int_{\frac{x}{1+x}}^{\frac{1}{1+x}} \frac{d\xi}{\xi}\log\frac{1-\xi}{\xi} = \frac{1}{2}\log^2 x - 2\log x\log(1+x) - 2Li_2(-x) - Li_2(-1) \quad (21b)$$

The "plus" distribution are defined on a test function G(x)

$$\int_0^1 dx\,[f(x)]_{+\,S.L.}\,G(x) \equiv \int_0^\infty dx\, f(x)\,\theta(1-x)\,[G(x)-G(1)] \quad (22a)$$

$$\int_1^\infty dx\,[f(x)]_{+\,T.L.}\,G(x) \equiv \int_0^\infty dx\, f(x)\,[\theta(x-1)\,G(x) + \theta(1-x)G(1)] \quad (22b)$$

Let us note that $R_{gg}(x)$ is the combination $R_{qq}(x) + R_{gq}(x) - R_{qg}(x)$, which appears in $P^o_{gg}(x)$ up to $\delta(1-x)$ terms.

With the $\bar{P}^{(1)}_{ij}(x)$ kernels (inverse Mellin transform of the time like cut vertices anomalous dimensions) and the $\bar{e}^{(1)}_j(x)$ functions (inverse Mellin transform of the time like cut vertices coefficient functions) we completed the list of necessary quantities for a next to leading logarithm analysis of scaling violations hard scattering processes.

In conclusion, we have shown that a kind of analytic continuation holds for the calculable quantities $P(x,\alpha)$ and $\bar{P}(x,\alpha)$ as well as for $e(x,\alpha)$ and $\bar{e}(x,\alpha)$ in Quantum Chromodynamics, as given by equation 5a,b. Equation 5b is similar to the relation proposed some years ago[13] for the structure function $F(x,q^2)$ and $\bar{F}(x,q^2)$ in the case of vanishing forward triple spectral function.

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LES EFFETS D'ORDRES SUPERIEURS POUR LES FONCTIONS DE STRUCTURE ET DE FRAGMENTATION EN CHROMODYNAMIQUE QUANTIQUE

- 1. La stratégie adoptée

Nous donnons dans ce chapitre, toutes les quantités nécessaires à l'analyse des violations de l'invariance d'échelle dans l'approximation d'ordre α^2. Il s'agit des fonctions coefficients $\overset{(\sim)}{E}_i^{\,F}(x,\alpha)$, de noyaux d 'évolution non singulet $\overset{(\sim)}{P}{}_{qq}^{NS}(x,\alpha;\pm)$ et singulet $\overset{(\sim)}{P}{}_{qq}^{S}(x,\alpha)$, $\overset{(\sim)}{P}_{qG}(x,\alpha)$, $\overset{(\sim)}{P}_{Gq}(x,\alpha)$, $\overset{(\sim)}{P}_{GG}(x,\alpha)$, ainsi que des noyaux d 'évolution valables pour les fonctions de structure non singulet $\overset{(\sim)}{K}{}^{NS}(x,\alpha;\pm)$ et des noyaux singulet $\overset{(\sim)}{K}_{FF}(x,\alpha)$, $\overset{(\sim)}{K}_{FG}(x,\alpha)$, $\overset{(\sim)}{K}_{GF}(x,\alpha)$, $\overset{(\sim)}{K}_{GG}(x,\alpha)$, qui interviennent dans les équations intégrodifférentielles valables pour les fonctions de structure (fragmentation) singulet :

$$\begin{pmatrix} \overset{\bullet}{\overset{(\sim)}{F}}{}^{S} \\ \overset{(\sim)}{\lambda}\,\overset{\bullet}{\overset{(\sim)}{G}} \end{pmatrix} = \begin{pmatrix} K_{FF} & K_{FG} \\ K_{GF} & K_{GG} \end{pmatrix} \begin{pmatrix} \overset{(\sim)}{F} \\ \overset{(\sim)}{\lambda}\,\overset{(\sim)}{G} \end{pmatrix} \qquad \text{avec} \quad \begin{matrix} \lambda = 1 \\ \tilde{\lambda} = -\frac{c_F}{2T_R} \end{matrix} \tag{1}$$

Les relations de réciprocité de Gribov et Lipatov d'ordre α ,

$$\tilde{K}^{o}_{FF}(x) = x\, K^{o}_{FF}\left(\tfrac{1}{x}\right) \tag{2a}$$

$$\tilde{\lambda}\,\tilde{K}^{o}_{FG}(x) = x\, K^{o}_{GF}\left(\tfrac{1}{x}\right) \tag{2b}$$

$$\tfrac{1}{\tilde{\lambda}}\,\tilde{K}^{o}_{GF}(x) = x\, K^{o}_{FG}\left(\tfrac{1}{x}\right) \tag{2c}$$

$$\tilde{K}^{o}_{GG}(x) = x\, K^{o}_{GG}\left(\tfrac{1}{x}\right) \tag{2d}$$

ne sont plus valables à l'ordre α^2 . Il en résulte que les corrections d'ordre α^2 sont beaucoup plus importantes dans le cas de l'annihilation que dans celui de la diffusion.

Les équations différentielles satisfaites par les quantités non-singulet et singulet ainsi exprimées, peuvent être résolues arithmétiquement par itération. Cela nous permet de trouver les fonctions de structure (fragmentation) et la fonction de distribution des gluons ($\overset{(\sim)}{F}{}_i^{NS}(x,Q)$, $\overset{(\sim)}{F}{}^{S}(x,Q)$, $\overset{(\sim)}{G}(x,Q)$), pour n'importe quelle valeur de Q, à partir de leurs valeurs à un certain Q_0 ($\overset{(\sim)}{F}{}_i^{NS}(x,Q_0)$, $\overset{(\sim)}{F}{}^{S}(x,Q_0)$, $\overset{(\sim)}{G}(x,Q_0)$). $G(x,Q_0)$ n'étant pas directement mesurable, on choisit, à défaut, une condition initiale sur la dérivée $\dot{F}^{S}(x,Q_0)$. Dans ce cas, $G(x,Q_0)$ peut être déterminé indirectement par l'équation:

$$G(x,Q_0) = K_{qG}^{-1}(x,\alpha(Q_0)) \otimes \left[\dot{F}^{S}(x,Q_0) - K_{FF}(x,\alpha(Q_0)) \otimes F^{S}(x,Q_0) \right] \quad (3)$$

Il en est de même pour l'annihilation ($\overset{(\sim)}{F}$, $\overset{(\sim)}{G}$, $\overset{(\sim)}{K}_{ij}$).

La relation supersymétrique:

$$\left[P_{qq}^{S} + P_{Gq} - P_{qG} - P_{GG} \right]_{C_F = 2T_R = C_A} = 0 \quad (4)$$

qui est vérifiée dans le cas où le schéma de renormalisation ne brise pas la supersymétrie, relie le coefficient de C_A^2 aux coefficients de $C_A C_F$, $C_F T_R$, T_R^2 , et $C_A T_R$.

Le nombre de quantités indépendantes diminue donc d'une unité.

Comme le nombre des diagrammes où intervient C_A^2 est très important, la relation supersymétrique simplifie beaucoup les calculs.

En utilisant cette relation et en déterminant le coefficient de C_A^2 , nous trouvons des résultats en accord avec ceux de la ref. [19] où ont été recalculées les dimensions anormales des opérateurs de twist minimum. Nous sommes par contre en désaccord avec les premiers calculs de dimensions anormales qui avaient été faits pour le coefficient de C_A^2 . [10]

Nous concluons que la relation supersymétrique et la continuation analytique ont permis d'éviter les erreurs possibles dans ces calculs difficiles, et de trouver les noyaux d'évolution pour la diffusion et l'annihilation, quantités très utiles pour l'analyse expérimentale.

HIGHER ORDER QCD EFFECTS IN INCLUSIVE ANNIHILATION AND DEEP INELASTIC SCATTERING

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Abstract : We present a parton model interpretation of the predictions of Quantum Chromodynamics in the process $e^+e^- \longrightarrow$ hadron + anything.

We give the complete list of parameters needed for the study of the scaling violations of fragmentation functions up to the next-to-leading logarithmic approximation. This includes flavour non-singlet and flavour singlet anomalous dimensions up to order α^2 and coefficient functions up to order α. We also present results for the deep inelastic scattering $e^- h \longrightarrow e^- + X$ anything.

We find that in e^+e^- annihilation the ratio of scaling violations of second order to first order is in general bigger than the corresponding ratio for deep inelastic scattering. The Gribov-Lipatov relation is thus violated in second order.

We also find that a modified Drell-Yan analytic continuation relation holds between the deep inelastic and annihilation structure functions for quarks and gluons.

In the N -space we give detailed numerical evaluation of the QCD effects for non-singlet and singlet densities, in the space-like and time-like regions.

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1. INTRODUCTION

During the past few years important progress has been made in understanding the prediction pattern of Quantum Chromodynamics for hard scattering and annihilation processes [1]. The emerging picture is that the colliding particle energy, stored in a very small region of the space time, is degraded by radiation of quarks and gluons which are transformed into hadrons when their space time separation becomes of the order of the hadronic size. This description leads to the factorization theorems for suitable averaged cross sections of hard processes [2]. These cross sections are thus expressed as products of two terms controlled by the short distance and large distance dynamics, respectively. The QCD predictability is based on the fact that the small scale dynamics is calculable in terms of a perturbative expansion in powers of the running coupling constant.

Among the different high energy processes, deep inelastic lepton nucleon scattering has, up to now, been the main place where perturbative predictions of QCD, including higher order and eventually part of higher twist corrections, have been tested [3] . The reason is first the fact that the q^2 range of data is larger in deep inelastic scattering, thus allowing a precise study of scaling violation. On the other hand the theoretical predictions based on local operator product expansion is possible only for these space-like processes. Thus the scaling violation in the time-like region had to use some other conjectured relation between structure and fragmentation functions, like the Gribov-Lipatov [4] or the analytic continuation [5] relations.

However the generalization of the Wilson expansion introduced by Mueller [6] can be applied both in the space-like and in the time-like region. Using this technique we present the complete list of quantities entering the description of scaling violation of one particle inclusive e^+e^- annihilation up to next-to-leading order. In section 2 we express the cut vertex formalism in a parton model language [7]. We thus can use a proper extension to all orders in α

of the Altarelli-Parisi [8] probabilistic interpretation. The fundamental calculable quantity we are concerned with is the evolution kernel which controls the scaling violation of the fragmentation functions. In section 3 we present the techniques for the calculation of the different parts of the evolution kernel, such as anomalous dimensions and coefficient functions for both the space-like ($eN \to e'X$) and time-like ($e^+e^- \to hX$) processes. As an example we will present, in some detail, the calculation of a two-loop diagram for both time and space-like cut vertices in appendix A.

In section 4 we present the results. In the non singlet case we have explicitly calculated all the quantities and we find, in the space-like region, expressions which agree with the known results [9]. In the singlet sector we explain how we can use the known space-like results [10] to obtain the time-like ones. In this section we only give the results in x space for 4 flavours and 3 colours. The general expressions, in moment and x space, are presented in appendix B. In section 5 we point out that the Gribov-Lipatov [4] reciprocity relation is not satisfied by the next-to-leading order and discuss the observed analytic relation [5] (or Drell, Levy, Yan) between space and time-like evolution kernels.

Finally section 6 is devoted to numerical estimates for the action of the evolution kernels. In particular when the kernels are applied to Gribov-Lipatov related structure functions, we compare the first and second order contributions to scaling violation in space and time-like regions.

2. PARTON MODEL INTERPRETATION OF QCD IN $e^+e^- \to h+X$:

Recently [6] the whole machinery of the renormalization group and Wilson operator product expansion has been extended to the time-like region and for almost all hard scattering processes. In the following we will use the results for the inclusive annihilation $e^+e^- \to h$ + anything and start with the factorization of the corresponding fragmentation functions [6, 11]

$$\bar{M}_i^\sigma(Q) \equiv \int_1^\infty dx\, x^{-\sigma-1} \bar{F}_i(x,Q) = \sum_\tau \bar{C}_{i,\tau}^\sigma(Q/\mu, \alpha(\mu))\, \bar{A}_\tau^\sigma \qquad (2.1)$$

Let us note the bar on the structure functions and their moments to remember the time-like region process. We use the variable $x = q^2/2P_h\cdot q$ instead of the usual $z = 1/x$ in order to keep the same variable as in deep inelastic scattering. In (2.1) i characterizes the Lorentz type of the fragmentation functions, $i = 2, L$ (or $i = 3$ for $W^\pm \longrightarrow h + X$) and $\tau = NS, q, g$ for non singlet, singlet quarks and gluons, respectively. In fact only singlet quarks and gluons mix and a suitable combination of fragmentation functions can isolate the non singlet piece like $\bar{F}_i^{NS, e^+e^-\to\pi} \equiv \bar{F}_i^{e^+e^-\to\pi^+} - \bar{F}_i^{e^+e^-\to\pi^0}$.

The time-like coefficient functions $\bar{C}_{i,\tau}^\sigma$ are calculable in perturbative expansion in $\alpha(\mu)$. The quantities $\bar{A}_\tau^\sigma$, which must be taken from experiment, represent the matrix elements between the observed hadron states of the time-like cut vertices, objects analogous to the Wilson operators of deep inelastic scattering. Using the renormalizability of the cut vertices [6] we can write (2.1) as

$$\begin{aligned}\bar{M}_i^\sigma(Q) = {} & \bar{C}_{i,NS}^\sigma(1,\alpha(Q))\, e^{-\int_0^t d\xi\, \bar{\gamma}_{NS}^\sigma(\alpha(\xi))} \cdot \bar{A}_{NS}^\sigma \\ & + \bar{C}_{i,\tau}^\sigma(1,\alpha(Q)) \left(T\, e^{-\int_0^t d\xi\, \bar{\gamma}_S^\sigma(\alpha(\xi))}\right)_{\tau\tau'} \cdot \bar{A}_{\tau'}^\sigma,\end{aligned} \qquad (2.2)$$

where now τ and τ' are restricted to q and g, $t = \frac{1}{2}\log Q^2/\mu^2$, $\bar{\gamma}_{NS}^\sigma$ are the anomalous dimensions of the $\bar{O}_{NS}^\sigma$ cut vertex and $(\bar{\gamma}_S^\sigma)_{\tau\tau'}$ the

anomalous dimension matrix elements of the singlet cut vertices $\bar{O}^{\sigma}_{\tau}$.

Taking the Mellin inverse of (2.2) one can write

$$\bar{F}_i(x,Q) = \bar{e}_{i,NS}(x,\alpha(Q)) \otimes D_{NS}(x,Q) + \bar{e}_{i,\tau}(x,\alpha(Q)) \otimes D_{\tau}(x,Q) \qquad (2.3)$$

where $\otimes$ means convolution and where we have defined

$$\int_1^{\infty} dx\ x^{-\sigma-1}\, \bar{e}_{i,\tau}(x,\alpha) = \bar{C}^{\sigma}_{i,\tau}(1,\alpha) \qquad , \tau = NS, q, g \qquad (2.4)$$

$$\int_1^{\infty} dx\ x^{-\sigma-1}\, D_{NS}(x,Q) = e^{-\int_0^t d\xi\, \bar{\gamma}^{\sigma}_{NS}(\alpha(\xi))} \cdot \bar{A}^{\sigma}_{NS} \qquad (2.5)$$

$$\int_1^{\infty} dx\ x^{-\sigma-1}\, D_{\tau}(x,Q) = \left(T e^{-\int_0^t d\xi\, \bar{\gamma}^{\sigma}_{S}(\alpha(\xi))} \right)_{\tau\tau'} \cdot \bar{A}^{\sigma}_{\tau'}, \qquad (2.6)$$

$\tau,\tau' = q, g.$

Now eq. (2.3) is not only a formal equation from which a renormalization group equation will give the evolution with Q but it has a parton model interpretation as proposed in ref. [7]. According to the factorization theorem we can think the process $e^+e^- \to \gamma^* \to h + X$ as being a two-step one. The time-like virtual photon decays first into quarks and gluons, $\gamma^* \to$ parton + X, and second the parton (quark or gluon) combines, with a lapse of much longer time, with surrounding partons to produce the observed hadrons, parton $\to h + X$. One can show (more easily in an axial gauge) that, with respect to the scaling variable $X > 1$ ($P_{parton} = X \cdot P_{hadron}$), the widths Γ(parton $\to h + X$), which are positive quantities in perturbation theory, coincide, after a suitable renormalization, with the quantities D_{NS} and D_{τ} defined in (2.5) and (2.6)

$$D_{NS}(x) = \Gamma(q_a \to h + X) - \Gamma(q_b \to h + X) \qquad (2.7)$$

where a and b are different flavour quark or antiquark indices

$$D_g(x) = \Gamma(g \to h + X) \tag{2.8}$$

$$D_q(x) = \sum_a \left(\Gamma(q_a \to h + X) + \Gamma(\bar{q}_a \to h + X)\right) . \tag{2.9}$$

In a similar way the width of the process $\gamma^* \to$ parton + X can be used to construct the quantities $\bar{e}_{i,z}(x,\alpha)$ defined in (2.4).

The important point about $D(x,Q)$ is that they satisfy the evolution equations to all orders in α which are directly deduced from (2.5) and (2.6) [12] :

$$Q\frac{\partial}{\partial Q} D_{NS}(x,Q) = \int_1^x \frac{dy}{y} \bar{P}_{NS}\left(\frac{x}{y}, \alpha(Q)\right) D_{NS}(y,Q) \tag{2.10}$$

$$= \bar{P}_{NS}(x, \alpha(Q)) \otimes D_{NS}(x,Q)$$

$$Q\frac{\partial}{\partial Q} D_z(x,Q) = \int_1^x \frac{dy}{y} \bar{P}_{zz'}\left(\frac{x}{y}, \alpha(Q)\right) D_{z'}(y,Q) \tag{2.11}$$

$$= \bar{P}_{zz'}(x, \alpha(Q)) \otimes D_{z'}(x,Q)$$

where the kernels $\bar{P}$ are the inverse Mellin transforms of the anomalous dimensions

$$\int_1^\infty dx\, x^{-\sigma-1} \bar{P}_{NS}(x,\alpha) = -\bar{\gamma}^{\sigma}_{NS}(\alpha) \tag{2.12}$$

$$\int_1^\infty dx\, x^{-\sigma-1} \bar{P}_{zz'}(x,\alpha) = -\left(\bar{\gamma}_S^{\sigma}(\alpha)\right)_{zz'} \tag{2.13}$$

Thus $\bar{P}_{\tau\tau'}(x)$ is the variation of the probability density of producing a parton τ' of momentum $p_{\tau'}$ out of the parton τ of momentum $p_\tau = x p_{\tau'}$, $x \geq 1$ (see fig. 1a). Such a variation will be compared to the space-like one $P_{\tau\tau'}(x)$ which is the variation of the probability density of finding, in a parton τ' of momentum $p_{\tau'}$, a parton τ of momentum $p_\tau = x p_{\tau'}$, $x \leq 1$ (see fig. 1b). Let us note that we have chosen the notation $\bar{P}_{\tau\tau'}(x)$ for $\bar{P}_{\tau\to\tau'}(x)$ and $P_{\tau\tau'}(x)$ for $P_{\tau\leftarrow\tau'}(x)$, [8], in order to have the same matrix notation in (2.11) and in the equivalent space-like evolution equation.

Although renormalization group invariant quantities, the cross sections D_{NS} and D_τ depend on the renormalization prescription for the cut vertices as the coefficient functions also do. So we have to consider evolution equations for physical (measurable) quantities like the fragmentation function $\bar{F}(x,Q)$. As mentioned before one can define non singlet fragmentation functions for which it is easy to write the evolution equation

$$Q\frac{\partial}{\partial Q}\bar{F}_{i,NS}(x,Q) = \left[\left(Q\frac{\partial}{\partial Q}\bar{e}_{i,NS}(\bar{\alpha})\right)\otimes \bar{e}^{-1}_{i,NS}(\bar{\alpha}) + \bar{P}_{NS}(x)\right]\otimes \bar{F}_{i,NS}(x,Q) \tag{2.14}$$

with $\bar{\alpha} = \alpha(Q)$ and where the inverse e^{-1} has to be understood with respect to the convolution as $e(x)\otimes e^{-1}(x) = \delta(1-x)$. The meaning of this equation is simple. Given the fragmentation function $\bar{F}(x,Q_0)$ at a given value of Q_0 for $x < x_0$, we can determine this fragmentation function for any Q value for $1 < x < x_0$ if we know $\alpha(Q)$.

In the singlet case, we cannot obtain a single equation for physical quantities. If we use two different singlet structure functions (for example $\bar{F}_{2,\,\text{singlet}}$ and $\bar{F}_{L,\,\text{singlet}}$ of the same experiment) we can write a system of two equations

$$Q\frac{\partial}{\partial Q}\bar{F}_{i,S} = \left[\left(Q\frac{\partial}{\partial Q}\bar{e}_S\right)\otimes\bar{e}_S^{-1} + \bar{e}_S\otimes\bar{P}_S\otimes\bar{e}_S^{-1}\right]_{ij}\otimes\bar{F}_{j,S}\,, \tag{2.15}$$

where $\bar{e}_s$ and $\bar{P}_s$ are the matrices

$$\bar{e}_s = \begin{pmatrix} \bar{e}_{2q} & \bar{e}_{2g} \\ \bar{e}_{Lq} & \bar{e}_{Lg} \end{pmatrix} \tag{2.16}$$

$$\bar{P}_s = (\bar{P}_{\tau\tau'}) \tag{2.17}$$

and $i,j = L,2$ and $\tau,\tau' = q,g$.

Equation (2.15) presents the practical difficulty that in present experiments the structure function $\bar{F}_L$ is usually badly measured. Another difficulty is that the inverse of the matrix $\bar{e}_s$ starts with $\frac{1}{\alpha}$ terms. So that to calculate the next-to-leading term in the evolution kernel of eq. (2.15) we need the α^2 contribution to the coefficient function $\bar{e}_s$ which has not been calculated up to now. In spite of this we find it interesting to study directly the QCD prediction for $\bar{F}_{L,\text{singlet}}$ even to the first order, since in recent experiments $\bar{F}_{L,\text{singlet}}$ is measured in different x and Q^2 regions. To first order we can write the evolution equations (2.15) as

$$\begin{pmatrix} \dot{F}_2 \\ \frac{4\pi}{\alpha}\dot{F}_L \end{pmatrix} = \frac{\alpha}{4\pi} \begin{pmatrix} K_{22} & K_{2L} \\ K_{L2} & K_{LL} \end{pmatrix} \otimes \begin{pmatrix} F_2 \\ \frac{4\pi}{\alpha} F_L \end{pmatrix} \tag{2.18}$$

where

$$K_{22} = 4C_F \left(\frac{1+x^2}{1-x} \right)_+ - 2C_F \left[2(1-x) + \delta(1-x) \right] \tag{2.19a}$$

$$K_{2L} = -x^2 \delta'(1-x) + 2 \tag{2.19b}$$

$$K_{L2} = 16 C_F^2 \left(x \log \frac{|1-x|}{x} + \frac{x}{2} \right)$$
$$+ 4 C_F C_A \left(-\frac{2}{x} + 4 - \frac{5}{3}x - 4x^2 - 4x \log \frac{|1-x|}{x^2} \right)$$
$$+ \frac{32}{3} C_F T_R \left(\frac{1}{x} - 3 + \frac{x}{2} + 2x^2 - 3x \log x \right) \qquad (2.19c)$$

$$K_{LL} = 8 C_A \left(\frac{1}{x} - 2 + x - x^2 + \frac{1}{(1-x)_+} \right)$$
$$+ 2 C_F \left(2(1-x) + \delta(1-x) \right) \qquad (2.19d)$$

In K_{2L}, which does not depend on any group factor, the derivative of $\delta(1-x)$ has to be understood in the distribution sense so that the moment of $x^2 \delta'(1-x)$ is $-(N+1)$.

For the above reasons we prefer to work with the method introduced by Baulieu and Kounnas, extended to all orders and define a coupled equation between the quantities $\bar{F}_2$ and D_g. This method avoids the $\frac{1}{\alpha}$ term because we have to take the inverse of $\bar{e}_{2q}$ only, which has a first non zero term. To be more precise, using (2.3) and (2.11) and writing D_q as

$$D_q = \bar{e}_{2q}^{-1} \otimes \bar{F}_2 - \bar{e}_{2q}^{-1} \otimes \bar{e}_{2g} \otimes D_g \qquad (2.20)$$

one obtains

$$Q \frac{\partial}{\partial Q} \begin{pmatrix} \bar{F}_2 \\ D_g \end{pmatrix} = \begin{pmatrix} K_{22} & K_{2g} \\ K_{g2} & K_{gg} \end{pmatrix} \otimes \begin{pmatrix} \bar{F}_2 \\ D_g \end{pmatrix} \qquad (2.21)$$

with

$$K_{22} = \left[Q \frac{\partial}{\partial Q} \bar{e}_{2q} + \bar{e}_{2q} \otimes \bar{P}_{qq} + \bar{e}_{2g} \otimes \bar{P}_{gq} \right] \otimes \bar{e}_{2q}^{-1} \qquad (2.22a)$$

$$K_{2g} = -\left(Q\frac{\partial}{\partial Q}\bar{e}_{2q} + \bar{e}_{2q}\otimes\bar{P}_{qq} + \bar{e}_{2g}\otimes\bar{P}_{gq}\right)\otimes\bar{e}_{2q}^{-1}\otimes\bar{e}_{2g} + Q\frac{\partial}{\partial Q}\bar{e}_{2g} + \bar{e}_{2g}\otimes\bar{P}_{gg} + \bar{e}_{2q}\otimes\bar{P}_{qg} \tag{2.22b}$$

$$K_{g2} = \bar{P}_{gq}\otimes\bar{e}_{2q}^{-1} \tag{2.22c}$$

$$K_{gg} = \bar{P}_{gg} - \bar{P}_{gq}\otimes\bar{e}_{2q}^{-1}\otimes\bar{e}_{2g} \; . \tag{2.22d}$$

The meaning of (2.21) is the following. Given the fragmentation function and its derivative at Q_o for $x < x_o$, first we can obtain $D_g(x,Q_o)$ and then solve the coupled system numerically starting from the initial values $\bar{F}_2(x,Q_o)$ and $D_g(x,Q_o)$. Let us note that in order to obtain D_g at Q_o from

$$D_g(x) = K_{2g}^{-1}\otimes\left(Q\frac{\partial}{\partial Q}\bar{F}_2 - K_{22}\otimes\bar{F}_2\right) \tag{2.23}$$

it is necessary for the term in α^2 in K_{2g} to give a smaller contribution than the α one in order to allow the inversion of the dominant term in $\bar{e}_{2q}\otimes\bar{P}_{qg}$ as given by Baulieu and Kounnas [13].

For deep inelastic processes, we have the same equations as (2.21) and (2.22) with $\bar{F}, \bar{e}$ and $\bar{P}$ changed into F, e, P and D_g changed into a gluon distribution.

3. CUT VERTICES AND THEIR COEFFICIENT FUNCTIONS :

The renormalization of the cut vertices has been discussed in the first papers on the subject [6] and explicit calculations have been done up to two loops in $[\varphi^3]_6$, [7, 14] and in QCD [15-19]. In ref. [11] it has been shown

that the factorization of mass singularities in the structure and fragmentation functions is a property equivalent to the renormalizability of cut vertices under the assumption of no additional infrared divergences. Bare cut vertices are non local operators and they are defined by their Feynman rules order by order in the coupling constant. Their renormalization properties are similar to those of the Wilson operators for deep inelastic scattering, which means that the flavour non singlet space or time-like cut vertices are multiplicatively renormalized, while under renormalization, the singlet ones mix together and with other operators [10].

It has been shown [7] that the renormalized space-like cut vertices are the analytic continuation in the spin of the renormalized Wilson operators. Although the proof is given for $[\varphi^3]_6$ the argument can be carried out for QCD too, because it depends on analytic properties of Feynman integrals and their discontinuities. In the case of gauge theories like QCD there is the following interesting difference. The structure functions for different lepton (electron or neutrino) scattering have different-crossing parities under the change $p \to p$, $q \to -q$ where p and q are the target and current momenta. So we have even (odd) momenta for even (odd) crossing parity structure functions. Now since we want to work with cut vertices in the x space and not in the N space, we have different possibilities for the analytic continuation of the moments in order to define their inverse Mellin transforms. The correct prescription is to use even (odd) moments and then extend the formulae analytically to complex moments for even (odd) crossing parity structure functions. The problem has been discussed in detail in space-like [20] and in time-like [15] regions.

We now give the Feynman rules for cut vertices time or space-like in the x space (in the moment space they are given in ref. [6]). We can easily obtain these rules from the diagrams of fig. 2 in the scaling limit $|q^2| \to \infty$ and $|q^2/2q\cdot k_i|$ fixed. In fig. 2 we present the Feynman rules for the flavour

non singlet cut vertices. The light like vector Δ_μ is chosen to have no transverse components in the frame where K_1 and q have no transverse components. This vector is employed to project out the definite Lorentz spin structure when the moments, with respect to the variable $x = |q^2/2p\cdot q|$, are taken (from zero to one in the space-like region, from one to infinity in the time-like one). The variables $\rho_\ell, \ell=1,2,3,$ are defined as

$$\rho_\ell = \Delta\cdot K_\ell / \Delta\cdot p \tag{3.1}$$

with

$$\Delta^2 = 0 \quad , \quad \Delta\cdot p > 0 \tag{3.2}$$

and where the indices i,j (a_1,a_2,a_3) are the quark (gluon) color indices.

The difference between space-like and time-like cut vertices lies in the orientation of the quark and gluon momenta. For example the quark momenta K_1 and K_2 in the space-like region are respectively incoming and outgoing while the inverse happens in the time-like region.

Now we come to the description of the calculation and we restrict ourselves for simplicity to the non singlet kernels. As it is evident from eq. (2.14), in order to calculate the next-to-leading order corrections to the evolution kernel, one has to calculate the first order coefficient functions $e(x,\alpha)$, $\bar{e}(x,\alpha)$ and the second order anomalous dimension $P(x,\alpha)$, $\bar{P}(x,\alpha)$. For the coefficient functions we had to calculate the cut diagrams of figs. (3a), (3b) and (3c) and the renormalized matrix elements of the first order cut vertices in figs.(3d) and (3e). For the second order (in α) anomalous dimension kernels we had to calculate the cut diagrams shown in fig. 4 and from these to extract their contribution to $P(x,\alpha)$ and $\bar{P}(x,\alpha)$. We did these calculations in the modified minimal subtraction scheme ($\overline{MS}$), working in the Feynman gauge of massless QCD. We had also to regularize possible infrared divergences coming from the zero mass of the quarks or gluons.

For this we used off-shell space (time) like external quark momenta for space (time) like cut vertices. For the regularization of the infrared divergences coming from the particular cuts of the diagrams, we employed dimensional regularizations as an infrared regulator, with a different regulator for each loop momentum integration. Since we are interested in the unpolarized structure or fragmentation functions we spin-averaged and projected out the definite spin objects (in the moment N space) by taking the trace of the cut vertex amplitude M, $\frac{1}{4\Delta p} \, tr \not{p} M$, where p is the external momentum. The Dirac algebra was done by using the algebraic language SCHOONSCHIP [21]. The result, a polynomial expression in the scalar products of the external and internal momenta and of the vector Δ_μ , was reduced, wherever possible, into linear combinations of propagators and external invariants. Thus we had to evaluate scalar integrals for different cuts of the corresponding diagram applying Cutkosky rules. In all cases it was possible to apply the following procedure. One of the subintegrals in every two-loop integration had only two propagators, cut or uncut, for which the integration could be done analytically. The final loop integration was made trivial after the first has been done. This was observed [9] for Wilson operators as the possibility of reduction of all overlapping divergences into non overlapping ones for the two-loop integration.

We had to evaluate different types of integrals according to the diagram and the particular cut we are looking for. In the calculation the following types of integrals appeared :

- Euclidean in both loop momenta (fig. 5a)
- one Euclidean and one cut loop momentum (fig. 5b)
- two cut loop momenta (fig. 5c).

All the integrals were performed by repeated application of the following simple formulae defined with the space-time dimension $n = 4-2\varepsilon$.

Space-like case $(p^2<0,\ \Delta\cdot p>0,\ 0<x<1)$

$$\{A,B\}_{S.L.} \equiv \int \frac{d^n k}{(2\pi)^n} \frac{\delta(x-\frac{\Delta\cdot k}{\Delta\cdot p})\,\theta(\frac{\Delta\cdot k}{\Delta\cdot p})\,f(\frac{\Delta\cdot k}{\Delta\cdot p})}{(-k^2-i0)^A}\ \mathrm{abs}\ \frac{1}{[-(p-k)^2-i0]^B} \tag{3.3}$$

$$= \frac{i}{(4\pi)^{2-\varepsilon}} \frac{\Gamma(A+B+\varepsilon-2)}{\Gamma(A)\Gamma(B)} (-p^2-i0)^{2-\varepsilon-A-B} f(x)\, x^{1-\varepsilon-A} (1-x)^{1-\varepsilon-B}$$

$$\{A,B\}^{\mu}_{S.L} \equiv \int \frac{d^n k}{(2\pi)^n} \frac{k^\mu \delta(x-\frac{\Delta\cdot k}{\Delta\cdot p})\,\theta(\frac{\Delta\cdot k}{\Delta\cdot p})\,f(\frac{\Delta\cdot k}{\Delta\cdot p})}{(-k^2-i0)^A}\ \mathrm{abs}\ \frac{1}{[-(p-k)^2-i0]^B} \tag{3.4}$$

$$= \{A,B\}_{S.L}\cdot x\cdot\left(p^\mu - \frac{p^2}{2\Delta p}\Delta^\mu\right) - \{A-1,B\}_{S.L}\cdot\frac{1}{x}\left(1+\frac{\varepsilon-1}{A-1}(1-x)\right)\frac{\Delta^\mu}{2\Delta\cdot p}$$

Time-like case $(p^2>0,\ \Delta p>0,\ 1<x<\infty)$

$$\{A,B\}_{T.L} \equiv \int \frac{d^n k}{(2\pi)^n} \frac{\delta(x-\frac{\Delta\cdot k}{\Delta\cdot p})\,f(\frac{\Delta\cdot k}{\Delta\cdot p})}{(-k^2-i0)^A}\ \mathrm{abs}\ \frac{1}{[-(p-k)^2-i0]^B} \tag{3.5}$$

$$= \frac{-i}{(4\pi)^{2-\varepsilon}} \frac{\Gamma(A+B+\varepsilon-2)}{\Gamma(A)\Gamma(B)} (-p^2-i0)^{2-\varepsilon-A-B} f(x)\, x^{1-\varepsilon-A} (1-x+i0)^{1-\varepsilon-B}$$

The time-like analogous relation to (3.4) can be obtained directly from (3.5).

Let us observe that eq. (3.5) for $x>1$ and $p^2>0$ is the analytic continuation of (3.3) defined for $x<1$ and $p^2<0$, up to an overall change of signs. We also point out that similar integrals appear in the calculation of Wilson operators

$$\{A,B\}_{W.O} \equiv \int \frac{d^n K}{(2\pi)^n} \frac{\delta(x-\frac{\Delta\cdot k}{\Delta\cdot p})\, f(\frac{\Delta\cdot k}{\Delta\cdot p})}{(-k^2-i0)^A} \frac{1}{[-(p-k)^2-i0]^B}$$

$$= \frac{i}{(4\pi)^{2-\varepsilon}} \frac{\Gamma(A+B+\varepsilon-2)}{\Gamma(A)\Gamma(B)} (-p^2-i0)^{2-\varepsilon-A-B} \int f(x)\, x^{1-\varepsilon-A} (1-x)^{1-\varepsilon-B} . \qquad (3.6)$$

Once the above relations are applied, one is left with convolution type integrals over the variables $x_1 = \frac{\Delta\cdot k_1}{\Delta\cdot p}$ and $x_2 = \frac{\Delta\cdot k_2}{\Delta\cdot p}$. The interesting point about this convolution structure is that it preserves the similarity between the relations (3.3), (3.4) and (3.5).

For the subtraction of the counterterms of the lower order we followed the method [22] in which one subtracts the counterterms in every scalar integral by removing the simple pole parts of the divergent subintegrals. Care must be taken in the numerator of the ε_1 and ε_2 dependence relative to the K_1 and K_2 loop integrations. The counterterms in this case are found directly from the diagrams by contraction of divergent subdiagrams and by looking at $\varepsilon_1, \varepsilon_2$ terms of the numerator in the resulting reduced diagrams [9,10].

In appendix A we present an example for the calculation of the contribution of a particular diagram to the space-like and time-like cut vertices.

4. THE EVOLUTION KERNELS :

In this section we present the evolution kernels of structure functions and fragmentation functions up to second order in the singlet and non singlet sector,

$$Q\frac{\partial}{\partial Q} \overset{(-)}{F}_{i,NS}(x,Q) = \overset{(-)}{K}_{i,NS}(x,a) \otimes \overset{(-)}{F}_{i,NS}(x,Q) \quad , i=2,3 , \qquad (4.1)$$

$$Q\frac{\partial}{\partial Q}\begin{pmatrix} \overset{(-)}{F}_{2,S}(x,Q) \\ \overset{(-)}{\lambda}\overset{(-)}{G}(x,Q) \end{pmatrix} = \begin{pmatrix} \overset{(-)}{K}_{22}(x,\alpha) & \overset{(-)}{K}_{2g}(x,\alpha) \\ \overset{(-)}{K}_{g2}(x,\alpha) & \overset{(-)}{K}_{gg}(x,\alpha) \end{pmatrix} \otimes \begin{pmatrix} \overset{(-)}{F}_{2,S}(x,Q) \\ \overset{(-)}{\lambda}\overset{(-)}{G}(x,Q) \end{pmatrix} \tag{4.2}$$

where $0 < x < 1$ in the space-like region (F, λG, K) and $1 < x < \infty$ in the time-like region ($\bar{F}$, $\bar{\lambda}\bar{G}$, $\bar{K}$). The factors $\overset{(-)}{\lambda}$ represent the relative normalization of $\overset{(-)}{F}_2$ and $\overset{(-)}{G}$ (see below).

As the kernels are distributions we decompose their convolution with any trial function $\Phi(x)$ as

$$K(x) \otimes \Phi(x) = K^{[1]}(x)\,\Phi(x) + \int_x^1 \frac{dy}{y}\,\Phi(y)\,K^{[2]}\left(\frac{x}{y}\right) + \int_x^1 \frac{dy}{y}\left[\Phi(y) - \frac{x}{y}\,\Phi(x)\right] K^{[3]}\left(\frac{x}{y}\right) \tag{4.3}$$

We take the same definition (4.3) in the space-like and time-like regions [f1]. The same ordering of the integral limits in (4.3) will avoid a change of signs when going from one region to another.

Let us note that, if the evolution of a structure or fragmentation function $\Phi(x)$ is given by a kernel $K(x)$, the evolution of the function

$$\Phi_K(x) \equiv x^K \Phi(x) \tag{4.4}$$

will be given by the kernel $K_K(x)$ defined in terms of $K(x)$ as

$$K_K(x) \otimes \Phi_K(x) = K^{[1]}(x)\,\Phi(x) + \int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^K \Phi_K(y)\,K^{[2]}\left(\frac{x}{y}\right) + \int_x^1 \frac{dy}{y}\left(\frac{x}{y}\right)^K \left[\Phi_K(y) - \left(\frac{x}{y}\right)^{1-K}\Phi_K(x)\right] K^{[3]}\left(\frac{x}{y}\right) \tag{4.5}$$

Thus the evolution of the experimental structure functions $x F_1^{exp}$, $x F_3^{exp}$ and F_2^{exp} can be directly obtained from eq. (4.5) with $k=1$ from the kernels $K^{[i]}$ given in this paper.

The general results are listed in appendix B and we restrict ourselves, in this section, to the $SU(3)_C$ case with 4 flavours.

4.1 The non-singlet sector

Using the cut vertices formalism presented in the previous section, we have computed the two-loop contribution to anomalous dimension in the space-like and time-like regions. In the space-like case our result agrees, diagram by diagram, with the known result [9]. This is not surprising as we expect such an agreement of cut vertices and OPE to all orders but it is a check of our computation.

The result in the time-like region appears such that the real part of its analytic continuation to $x<1$ is nothing else but the space-like result. In fact this is not only true for the contribution of each diagram but also for the contribution of each integral, once all the relevant cuts have been taken into account. The signification of this analytic continuation will be discussed in detail in the following section. However let us insist on how powerful this result is in going from one region to the other.

As the same analytic continuation is true for the coefficient functions, all the kernels satisfy

$$K^{[i]}(x) = \text{Real part (analytic continuation of } \bar{K}^{[i]}(x) \text{ to } x<1 \text{)} . \qquad (4.6)$$

Let us remind that there is no minus sign in front of the right-hand part of (4.6) because it has been absorbed in the limits of integration in (4.3).

The detailed result is given in appendix B and we quote here the non singlet kernel in the case of $SU(3)_C$ with 4 flavours which applies to

$\bar{F}_2^{e^+e^-\to\pi^+} - \bar{F}_2^{e^+e^-\to\pi^0}$, $\bar{F}_2^{e^+e^-\to p} - \bar{F}_2^{e^+e^-\to n}$ or F_2^{ep-en} with $\eta=+1$ and to $F_2^{\nu N} - F_2^{\bar{\nu}N}$ with $\eta=-1$

$$K_{2,NS}^{(-)[1]} = \frac{\alpha}{4\pi}\left[8+\frac{32}{3}\log|1-x|\right] + \left(\frac{\alpha}{4\pi}\right)^2\left[433.8714+152.2057\log|1-x| - \frac{400}{9}\,\mathrm{Re}\left(\log^2(x-1)+\pi^2\right)\right], \tag{4.7a}$$

$$K_{2,NS}^{(-)[2]} = -\frac{\alpha}{4\pi}\frac{16}{3}(1+x) + \left(\frac{\alpha}{4\pi}\right)^2\cdot\frac{16}{9}\cdot\Big[-\frac{1}{3}(194+503x + 12\eta(1-x)) + 2(3-5x-\eta(1+x))\log x + (1+x)\cdot(25\log|1-x| - 4\log^2 x + 3\pi^2) + \frac{1+x^2}{1-x}\log x\,(9\log x + 38 - 16\log|1-x|) - 2\eta\frac{1+x^2}{1+x}\left(\frac{1}{2}\log^2 x - 2\log x\cdot\log(1+x) - 2\,Li_2(-x) - \frac{\pi^2}{6}\right)\Big], \tag{4.7b}$$

$$K_{2,NS}^{(-)[3]} = \frac{\alpha}{4\pi}\frac{32}{3}\cdot\frac{1}{1-x} + \left(\frac{\alpha}{4\pi}\right)^2\left(152.2057 - \frac{800}{9}\log(1-x)\right)\frac{1}{1-x}, \tag{4.7c}$$

where $Li_2(x)$ is the dilogarithm function

$$Li_2(x) = -\int_0^x dy \, \frac{\log(1-y)}{y} \tag{4.8}$$

For completeness let us quote $K_{3,NS}$ which applies to $F_3^{\nu-\bar{\nu}}$ with $\eta=+1$ and $F_3^{\nu+\bar{\nu}}$ with $\eta=-1$

$$K_{3,NS}^{(-)[1]} = K_{2,NS}^{(-)[1]} \tag{4.9a}$$

$$K_{3,NS}^{(-)[2]} = K_{2,NS}^{(-)[2]} + \left(\frac{\alpha}{4\pi}\right)^2 \frac{400}{9}(1+x) \tag{4.9b}$$

$$K_{3,NS}^{(-)[3]} = K_{2,NS}^{(-)[3]} \tag{4.9c}$$

The analytic continuation from $x>1$ to $x<1$ of the expressions which appear in (4.7) and (4.9) is trivial except for $Re\left[\log^2(x-1)+\pi^2\right]$ in (4.8a) which is $\log^2(x-1)+\pi^2$ for $x>1$ and $\log^2(1-x)$ for $x<1$. This term which comes from the coefficient function also appears in some diagrams contributing to anomalous dimension but is cancelled in the total anomalous dimension. Its importance in time-like region will be discussed in section 6.

The time-like result of the evolution kernel (4.7) agrees with the result of ref. [15] although the separate results for the anomalous dimension and the coefficient function, respectively differ due to a different choice of renormalization prescription.

4.2 The singlet sector

We have pointed out that the analytic continuation we obtained for the non singlet anomalous dimension was in fact true for each integral which appears in the two-loop diagram, when summing over all the cuts. As the

integrals in the OPE calculation of the singlet anomalous dimension are the same as those of the non-singlet case [10], the analytic continuation we have proved in non-singlet case also applies in the singlet sector. However we must take into account the crossing matrix for the colour, flavour and spin degrees of freedom [26] . This can be done either by taking different normalization factors λ and $\bar{\lambda}$ in eq. (4.2), or by absorbing their ratio in the non-diagonal element of the evolution kernel. In order to exploit, in a maximal way, the simple analytic continuation, eq. (4.6), discussed above, we choose the first alternative with $\lambda = 1$, $\bar{\lambda} = -C_F/2T_R$. In this way the time-like kernel can be obtained from the known results in the space-like region [10] by performing the analytic continuation (4.6). To this end, we can easily make the inverse Mellin transform of the existing compact form of the moments [23][f5]. In ref. [24] using a quite plausible relation between the four elements of the singlet anomalous dimension matrix for the case of supersymmetric representation of quarks, $C_F = C_A = 2T_R$, the correct coefficient of C_A^2 in the matrix element $P_{gg}(x)$ was obtained, which agrees with that of ref. [19]. So below, using this corrected result for $P_{gg}(x)$, we give the matrix kernel for the coupled evolution equation (2.21) between the structure or fragmentation F_2 ($\bar{F}_2$) and the gluon distribution or fragmentation D_g is given, as the non-singlet case, for $SU(3)_C$ with 4 flavours and applies to

$\bar{F}_2^{e^+e^-\to\pi^+} + \bar{F}_2^{e^+e^-\to\pi^0}$, $\bar{F}_2^{e^+e^-\to p} + \bar{F}_2^{e^+e^-\to n}$, F_2^{ep+en} or $F_2^{\nu N+\bar{\nu}N}$

$$\overset{(-)}{K}{}_{22}^{[1]} = \frac{\alpha}{4\pi}\cdot\left(8+\frac{32}{3}\log|1-x|\right) + \left(\frac{\alpha}{4\pi}\right)^2\cdot\left[433.8714 + 152.2057\log|1-x| - \frac{400}{9}\,\mathrm{Re}\left(\log^2(x-1)+\pi^2\right)\right] \qquad (4.10a)$$

$$\overset{(-)}{K}{}_{22}^{[2]} = -\frac{\alpha}{4\pi}\frac{16}{3}(1+x) + \left(\frac{\alpha}{4\pi}\right)^2\frac{16}{9}\left[\frac{1}{3}\left(\frac{208}{x}-830-11x-64x^2\right) + 4(1-3x+24x^2)\log x - 4(1+x)\left(12\,\mathrm{Re}\,Li_2(x)+13\log^2 x-\frac{11}{4}\pi^2\right)\right.$$

$$+\left(\frac{32}{x}+49+x-32x^2\right)\log|1-x|-2\frac{1+x^2}{1+x}F(x)$$
$$+\frac{1+x^2}{1-x}\log x\left(38+9\log x-16\log|1-x|\right) \tag{4.10b}$$

$$\overset{(-)}{K}{}^{[3]}_{22}=\frac{\alpha}{4\pi}\frac{32}{3}\frac{1}{1-x}$$
$$+\left(\frac{\alpha}{4\pi}\right)^2\left(152.2057-\frac{800}{9}\log|1-x|\right)\frac{1}{1-x} \tag{4.10c}$$

$$\overset{(-)}{K}{}^{[1]}_{2g}=0 \tag{4.11a}$$

$$\overset{(-)}{K}{}^{[2]}_{2g}=\frac{\alpha}{4\pi}16(1-2x+2x^2)+\left(\frac{\alpha}{4\pi}\right)^2\frac{16}{3}\Big[\frac{52}{x}-157-304x+423x^2$$
$$+2(2-208x+201x^2)\log x-(41+134x+2x^2)\log^2 x$$
$$+2\left(\frac{12}{x}-13+156x-177x^2\right)\log|1-x|-4(13+28x+$$
$$+8x^2)\left(\mathrm{Re}\,Li_2(x)-\frac{\pi^2}{6}\right)-2(1-2x+2x^2)(26\log x\log|1-x|$$
$$-10\log^2|1-x|+\frac{17\pi^2}{6}+\left(\mathrm{Re}\log^2(x-1)+\pi^2\right)+$$
$$+18(1+2x+2x^2)F(x)\Big] \tag{4.11b}$$

$$\overset{(-)}{K}{}^{[3]}_{2g}=0 \tag{4.11c}$$

$$\overset{(-)}{K}{}^{[1]}_{g2}=0 \tag{4.12a}$$

$$K_{g2}^{(-)[2]} = \frac{\alpha}{4\pi}\frac{16}{3}\left(\frac{2}{x}-2+x\right)+\left(\frac{\alpha}{4\pi}\right)^2\frac{32}{9}\left[-\frac{1}{3}\left(\frac{53}{x}-125-41x-132x^2\right)\right.$$

$$-(100+37x+24x^2)\log x+\frac{9}{2}\left(\frac{2}{x}+6+3x\right)\log^2 x+ \qquad (4.12b)$$

$$+\left(\frac{50}{x}-58+37x\right)\log|1-x|-\left(\frac{2}{x}-2+x\right)\cdot(10\log x\cdot\log|1-x|$$

$$+\frac{\pi^2}{6}-\mathrm{Re}\left(\log^2(x-1)+\pi^2\right)+4\left(\frac{4}{x}-2+x\right)\left(\mathrm{Re}\,Li_2(x)-\frac{\pi^2}{2}\right)$$

$$\left.-9\left(\frac{2}{x}+2+x\right)F(x)\right]$$

$$K_{g2}^{(-)[3]} = 0 \qquad (4.12c)$$

$$K_{gg}^{(-)[1]} = \frac{\alpha}{4\pi}\left(\frac{50}{3}+24\log|1-x|\right)+\left(\frac{\alpha}{4\pi}\right)^2\left[366.3110+\right. \qquad (4.13a)$$

$$\left.+192.4628\log|1-x|\right]$$

$$K_{gg}^{(-)[2]} = \frac{\alpha}{4\pi}24\left(\frac{1}{x}-2+x-x^2\right)+\left(\frac{\alpha}{4\pi}\right)^2 8\left[-\frac{1}{18}\left(\frac{552}{x}-\right.\right.$$

$$-343+1117x-360x^2)-3\left(\frac{1}{x}-2+x-x^2\right)\pi^2$$

$$-\frac{1}{9}(843-225x+1252x^2)\log x+9\left(\frac{1}{x}+2+5x-x^2+\right.$$

$$\left.+\frac{1}{1-x}\right)\log^2 x-\frac{16}{9}\left(\frac{4}{x}+3-3x-4x^2\right)\log|1-x|$$

$$+\frac{32}{3}(1+x)\left(\mathrm{Re}\,Li_2(x)-\frac{\pi^2}{6}\right)-36\left(\frac{1}{x}-2+x-x^2+\frac{1}{1-x}\right) \qquad (4.13b)$$

$$\left.*\log x\log|1-x|-18\left(\frac{1}{x}+2+x+x^2-\frac{1}{1+x}\right)F(x)\right]$$

$$K^{(-)\,[3]}_{gg} = \frac{\alpha}{4\pi}\frac{24}{1-x} + \left(\frac{\alpha}{4\pi}\right)^2 \frac{192.4629}{1-x} \qquad (4.13c)$$

where

$$\mathrm{Re}\,Li_2(x) = -\int_0^x dy\,\frac{\log|1-y|}{y}\,, \qquad (4.14)$$

$$F(x) = \frac{1}{2}\log^2 x - 2\,Li_2(-x) - 2\log x \log(1+x) - \frac{\pi^2}{6}\,. \qquad (4.15)$$

Let us point out that the analytic continuation (4.6) is no more true for the singlet kernel because as we have seen it is true for the coefficient functions e and anomalous dimension P, respectively, while in the matrix singlet kernel (2.23) the convolution of the real parts appears and not the real part of the convolution of the analytically continued e and P.

In the time-like region we cannot compare our result with other calculations [19] because the authors do not give the singlet coefficient functions which are necessary to form a physical quantity.

5. RELATION BETWEEN DEEP INELASTIC AND ANNIHILATION STRUCTURE FUNCTIONS

Since the early days of the parton model the connection between the structure functions of the crossing related processes, $e^- h \to e^- + X$ and $e^+e^- \to \bar{h} + X$ was studied. The first relation, called analytic continuation or Drell-Levy-Yan relation, was found in cut-off field theories [5] [f2] :

$$\bar{F}(x) = -F(x) \qquad (5.1)$$

where $F(x)$ and $\bar{F}(x)$ are the supposedly existing Bjorken limits $\left(x = |q^2/2pq|\right.$

fixed and $q^2 \to \infty$) of the structure function $F(x,q^2)$ of deep inelastic and of the fragmentation function $\bar{F}(x,q^2)$ of inclusive annihilation. In eq. (5.1) the variable x has the values $0 < x < \infty$, $x \neq 1$. Further study [25] of the above relation showed that in perturbation theory eq. (5.1) is wrong due to the discontinuities of $F(x,q^2)$ for $x > 1$ and contributions from multiple spectral functions. In generalized ladder diagrams eq. (5.1) was modified into :

$$\bar{F}(x) = - Re\, F(x) + \rho(x) \qquad (5.2)$$

where Re refers to the real part of $F(x)$ and $\rho(x)$ comes from multiple spectral functions.

With the appearance of the renormalization group methods (RG) it was possible to study the analytic continuation problem in the leading logarithmic approximation [4] . In the language of RG the problem is reduced to the study of analytic properties of the coefficient functions $e(x,\alpha)$, $\bar{e}(x,\alpha)$ and of the anomalous dimension kernels $P(x,\alpha)$, $\bar{P}(x,\alpha)$. For QCD this problem was studied recently in the leading log approximation in ref. [26].

Another relation (called Gribov-Lipatov relation) between the structure functions $F(x,q^2)$ and $\bar{F}(x,q^2)$ was found to hold in the leading logarithmic approximation for various field theories in refs.[4, 27] specially for QCD in ref. [26] . This relation states that [f3] :

$$\bar{F}_i\left(\frac{1}{x}, q^2\right) = \frac{1}{x^{\lambda_i}} F_i(x,q^2) \quad , \qquad 0 < x < 1 \qquad (5.3)$$

where λ_i depends on the dimensions d of the space time and on the kind of the structure function (F_1, F_2) ; for $d = 4$, $\lambda_1 = 1$, $\lambda_2 = 3$

The Gribov-Lipatov relation is translated, in the language of renormalization group methods, into properties of coefficient functions and anomalous dimensions (see later).

For the evolution equation of sect. 2 the Gribov-Lipatov and analytic

continuation relation reflect properties for the evolution kernel as eqs (2.14) and (2.22).

For the non-singlet kernel :

$$K_{NS} = Q\frac{\partial}{\partial Q} e_{NS} \otimes e_{NS}^{-1} + P_{NS}(x,\alpha)$$

analytic continuation rel. (5.2) reads :

$$K_{NS}(x,\alpha) = -\,Re\,\bar{K}_{NS}(x,\alpha) \qquad x<1 \tag{5.4}$$

and Gribov-Lipatov relation :

$$\bar{K}_{NS}(x,\alpha) = x\,K_{NS}\left(\frac{1}{x},\alpha\right). \tag{5.5}$$

Eq. (5.4) is not a well-defined relation between the two kernels K_{NS} and $\bar{K}_{NS}$ because these kernels are distribution functions which subtract singularities at the point $x=1$. So, one needs a continuation relation which would be valid for distribution functions. We note that this is true even to the first order in α where the kernels $\bar{K}$ and K are the well-known Altarelli-Parisi kernels. Because we use dimensional regularization to regularize the $x=1$ singularity (soft-gluon emission) the different distributions are obtained in a dimensionally regularized representation. For example the sum of the cuts of fig. (6) gives terms which are nothing else than the dimensional representation form of the $\frac{1}{(1-x)_+}$ distribution.

$$f(x) = \lim_{\varepsilon\to 0}\left[\frac{1}{(1-x)^{1-\varepsilon}} - \frac{1}{\varepsilon}\,\delta(1-x)\right], \qquad x\le 1 \tag{5.6}$$

The first part of this form, $\frac{1}{(1-x)^{1-\varepsilon}}$ comes from the real emission of a gluon and the second one $-\frac{1}{\varepsilon}\delta(1-x)$ is the virtual contribution.

The corresponding diagram for the time-like case gives

$$\bar{f}(x) = \lim_{\varepsilon\to 0}\left[\frac{1}{(x-1)^{1-\varepsilon}} - \frac{e^{i\pi\varepsilon}}{\varepsilon}\,\delta(1-x)\right], \qquad x\ge 1 \tag{5.7}$$

Here also the first part comes from the real emission of a gluon and the second one from the virtual gluon contribution.

We want to show, with a simple example, how the naïve limits for $\varepsilon \to 0$, in the formulae (5.6-5.7), define the distributions $\frac{1}{(1-x)_+}$ and $\frac{1}{(x-1)_+}$, respectively. It is well known that the expressions $\frac{1}{(1-x)^{1-\varepsilon}}$ and $\frac{1}{(x-1)^{1-\varepsilon}}$, respectively, can be used to define distributions through the coefficient of their Laurent expansions in ε [28]

$$\frac{1}{(1-x)^{1-\varepsilon}} = \frac{1}{\varepsilon}\,\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left(\frac{\log(1-x)}{1-x}\right)_+ + \cdots \,, \quad 0<x<1 \tag{5.8}$$

$$\frac{1}{(x-1)^{1-\varepsilon}} = \frac{1}{\varepsilon}\,\delta(1-x) + \frac{1}{(x-1)_+} + \varepsilon\left(\frac{\log(x-1)}{x-1}\right)_+ + \cdots \,, \quad 1<x \tag{5.9}$$

We see that if we substitute (5.8) in (5.6) and (5.9) in (5.7) the pole terms cancel and we obtain :

$$f(x) = \lim_{\varepsilon\to 0}\left[\frac{1}{(1-x)_+} + \varepsilon\left(\frac{\log(1-x)}{1-x}\right)_+ + \cdots\right], \quad 0<x<1 \tag{5.10}$$

$$\bar{f}(x) = \lim_{\varepsilon\to 0}\left[\frac{1}{(x-1)_+} - i\pi\,\delta(1-x) + \varepsilon\left(\left(\frac{\log(x-1)}{x-1}\right)_+ + \frac{\pi^2}{2}\,\delta(1-x)\right) + \cdots\right], \tag{5.11}$$

$x > 1$.

The S.L. cut vertex gives a real contribution for both of the cuts contrary to the T.L. one which gives an imaginary part with infrared singularity of the type $i\pi\delta(1-x)$ but which cancels with the symmetric diagram contribution (complex conjugate diagrams). So in the case of T.L. cut vertices the cancellation of infrared divergences does not take place diagram by diagram after the sum of all the cuts, but we have to add the complex conjugate diagrams. In the two-loop calculation the coefficients of the linear and the

quadratic terms in ε , eqs.(5.10-5.11), appear in the contribution of particular diagrams (this does not appear in the axial gauge [15]). On the other hand none of these terms appear in the total sum of all the diagrams. On the contrary, in the calculation of the coefficient functions the term $\left(\frac{\log(1-x)}{(1-x)}\right)_+$ remains. The analytic continuation of this term gives the famous π^2 of the time-like region.

In sect. 2 we decomposed the convolution of the distribution kernels, $K(x)$, with the structure functions into three terms each of which is the integral of regular functions $K^{[i]}(x)$, $i=1,2,3$, times the structure functions. In this way the numerical work was made easier and the numerical approximations controllable.

Another important simplification which occurs with the use of these regular kernels, is that, the analytic continuation relation (5.4) is now well defined :

$$K^{[i]}(x) = \text{Real part } [\,\bar{K}^{[i]}(x)\,] \quad , \; i=1,2,3 \; , \quad 0<x<1 \; . \tag{5.12}$$

Now let us present the form of the Gribov-Lipatov (G.L.) relation and analytic continuation (A.C.) relation in the space of moments. When the G.L. relation holds we find

$$K_\sigma = \bar{K}_{\sigma+1} \tag{5.13}$$

where

$$K_\sigma \equiv \int_0^1 dx \; x^{\sigma-1} K(x) \tag{5.14a}$$

$$\bar{K}_\sigma \equiv \int_1^\infty dx \; x^{-\sigma-1} \bar{K}(x) \tag{5.14b}$$

In the case when A.C. relation holds there is not a simple relation for the moments. For example, terms coming from derivatives , $\frac{\partial}{\partial a}$, of the monomial

x^a , or their convolutions, respect analytic continuation and their moments

$$\frac{\partial^N}{\partial a^N}\frac{1}{\sigma+a} = \frac{\partial^N}{\partial a^N}\int_0^1 dx\; x^{\sigma-1}x^a , \tag{5.15a}$$

$$\frac{\partial^N}{\partial a^N}\frac{1}{-\sigma+a} = \frac{\partial^N}{\partial a^N}\int_1^\infty dx\; x^{-\sigma-1}x^a \tag{5.15b}$$

are connected by the substitution $\sigma\to-\sigma$.

On the other hand the moments of analytically continued terms like $\frac{1}{(1-x)_+}$ and $\frac{1}{(x-1)_+}$ are connected by the change $\sigma\to\sigma+1$. Up to the next-to-leading order all the terms which appear in the kernel can be derived from the following generic forms (or from their convolutions),

$$x^a(1-x^K)^{-1+\delta} - \frac{1}{K\cdot\delta}\delta(1-x) \quad , \quad 0<x<1 \tag{5.16}$$

or

$$x^a(x^K-1)^{-1+\delta} - \frac{1}{K\cdot\delta}e^{i\pi\delta}\delta(1-x) \quad , \quad x>1 . \tag{5.17}$$

where $K=1,2$. These terms generalize the expressions in eqs (5.6-5.7). Let us take the moments in the space-like region $0<x<1$ [29] .

$$\begin{aligned} K_\sigma &\equiv \int_0^1 dx\; x^{\sigma-1}\left[x^a(1-x^K)^{-1+\delta} - \frac{1}{K\cdot\delta}\delta(1-x)\right] \\ &= \frac{1}{K}B\left(\frac{\sigma+a}{K},\delta\right) - \frac{1}{K\delta} \\ &= \frac{1}{K}\left[\psi(1)-\psi\left(\frac{\sigma+a}{K}\right) + \frac{\delta}{2}\left(\left(\psi(1)-\psi\left(\frac{\sigma+a}{K}\right)\right)^2 + \right.\right. \\ &\quad \left.\left. + \psi'(1) - \psi'\left(\frac{\sigma+a}{K}\right)\right) + O(\delta^2)\right] , \end{aligned} \tag{5.18}$$

where

$$\psi^{(m)}(z) \equiv \frac{d^{m+1}}{dz^{m+1}} \log \Gamma(z) \quad , \; m = 0, 1, 2, \ldots$$

the poly-Gamma functions.

The corresponding moments in the time-like region $1 < x < \infty$ are, [29]

$$\bar{K}_\sigma \equiv \int_1^\infty dx \; x^{-\sigma-1} \left[x^a (x^\kappa - 1)^{-1+\delta} - \frac{e^{i\pi\delta}}{\kappa \cdot \delta} \delta(1-x) \right]$$

$$= \frac{1}{\kappa} B\left(\frac{\sigma-a}{\kappa} - \delta + 1, \delta\right) - \frac{e^{i\pi\delta}}{\kappa\cdot\delta} \tag{5.19a}$$

$$= \frac{1}{\kappa} \left[\frac{\sin(\pi \frac{a-\sigma}{\kappa})}{\sin \pi (\delta + \frac{a-\sigma}{\kappa})} B\left(\frac{a-\sigma}{\kappa}, \delta\right) - \frac{e^{i\pi\delta}}{\delta} \right] \tag{5.19b}$$

$$= \frac{1}{\kappa} \left[\psi(1) - \psi\left(\frac{\sigma-a}{\kappa}+1\right) - i\pi + \frac{\delta}{2} \left(\left(\psi(1) - \psi\left(\frac{\sigma-a}{\kappa}+1\right)\right)^2 \right.\right.$$

$$\left.\left. + \pi^2 + \psi'(1) + \psi'\left(\frac{\sigma-a}{\kappa}+1\right) \right) + O(\delta^2) \right] \tag{5.19c}$$

In equation (5.19b) we see that the analytic continuation $\sigma \to -\sigma$ is almost true apart from the factor $\sin \pi \frac{(a-\sigma)}{\kappa} \Big/ \sin \pi \left(\frac{a-\sigma}{\kappa} + \delta\right)$, which is necessary to kill the singularities (simple poles) when $\frac{a-\sigma}{\kappa}$ is a negative integer. Equation (5.19c), which comes from the expansion of (5.19a), suggests that going from the time-like to the space-like region we should make the replacement

$$\psi^{(m)}\left(\frac{\sigma-a}{\kappa}+1\right) \to (-1)^m \psi^{(m)}\left(\frac{\sigma+a}{\kappa}\right) + i\pi \delta_{m,0} \quad , \; m = 0, 1, 2, \ldots \tag{5.20}$$

and take the real part of the result.

Using the relation [29],

$$\psi^{(m)}(z) - (-1)^m \psi^{(m)}(1-z) = -\pi \frac{d^m}{dz^m} \operatorname{ctg} \pi z \quad , \; m = 0, 1, 2, \ldots \tag{5.21}$$

we see that another way of expressing the substitution (5.20), which is closer to the rule $\sigma \to -\sigma$, is the following one,

$$\psi^{(m)}\left(\frac{a-\sigma}{\kappa}\right) \to (-1)^m \psi^{(m)}\left(\frac{\sigma+a}{\kappa}\right) - \pi \frac{d^m}{dz^m} \operatorname{ctg} \pi z \Big|_{z=\frac{a-\sigma}{\kappa}} + i\pi \delta_{m,0} . \quad (5.22)$$

Relation (5.22) is the key to pass from the space-like to the time-like region. Indeed it gives the regularization procedure when $N \to -N$ for the quantities appearing in the evolution kernels. The set of substitutions we need is the following (see Appendix B),

$$N \to -N \quad (5.23a)$$

$$S_1(N) \to S_1(N-1) \quad , \quad S_1^2(N) \to S_1^2(N-1) + \pi^2 \quad (5.23b)$$

$$S_2(N) \to -S_2(N-1) + \frac{\pi^2}{3} \quad , \quad S_2(N) - \frac{1}{2} S_2'\left(\frac{N}{2}\right) \to S_2(N-1) - \frac{1}{2} S_2'\left(\frac{N-1}{2}\right) \quad (5.23c)$$

$$S_3(N) \to S_3(N-1) \quad , \quad 2\tilde{S}(N) - \frac{1}{4} S_3'\left(\frac{N}{2}\right) \to 2\tilde{S}(N-1) - \frac{1}{4} S_3'\left(\frac{N-1}{2}\right) \quad (5.23d)$$

Now we come to an illustration of the violation of the G.L. relation to next-to-leading order in the evolution kernel. We give the expression for $F_{2,NS}$ and for the other structure functions the G.L. violation can be found from the results in appendix B

$$\begin{aligned} \Delta K^{(1)}_{NS,2}(x) &\equiv K^{(1)}_{NS,2}(x) - x\, \bar{K}^{(1)}_{NS,2}\left(\frac{1}{x}\right) \\ &= -8 C_F^2 \left[\left(-5 - x + \frac{6}{1-x}\right) \log x + \left(3 + 3x - \frac{4}{1-x}\right) \log^2 x \right. \\ &\quad \left. + 4 \frac{1+x^2}{1-x} \log x \log|1-x| \right] + 2 b_0 C_F \left[-6 \frac{1+x^2}{1-x} \log x \right. \\ &\quad \left. + 7(1+x) - 2\pi^2 \delta(1-x) \right] \end{aligned} \quad (5.24)$$

or in moment space

$$\Delta K^{(1)}_{2,NS}(N) = 16 C_F^2 \left[2 S_1(N) \cdot \frac{2N+1}{N^2(N+1)^2} - \left(2S_1(N) - \frac{1}{N(N+1)} - \frac{3}{2} \right) \cdot \right.$$
$$\left. \cdot \left(2 S_2(N) - \frac{\pi^2}{3} \right) - \frac{1}{2} \frac{6N^3 + 9N^2 + 7N + 2}{N^3(N+1)^3} \right] +$$
$$+ 2 b_0 C_F \left[12 S_2(N) - \frac{2N+1}{N^2(N+1)^2} \cdot (7N^2 + 7N + 6) \right] \qquad (5.25)$$

6. NUMERICAL COMPARISON OF SPACE AND TIME-LIKE SCALING VIOLATIONS

If the next-to-leading log contribution definitely violates the Gribov-Lipatov reciprocity relation, it remains to estimate how big such a violation is. For the sake of simplicity we will first consider numerical estimation in the non-singlet case of $SU(3)_c$ with 4 flavours, and compare first and second order contributions in the space and time-like regions both in moment and in x - space (see Appendix B for detailed expressions).

6.1 The non-singlet case

In the moment space, the space-like second order effect can be summarized by equation [9]

$$F_N(Q^2) \equiv \int_0^1 dx\, x^{N-1} F(x, Q^2) = A_N \cdot R_N(Q^2) \left[\alpha_0(Q^2) \right]^{-\frac{\gamma_0^N}{2b_0}} \qquad (6.1)$$

where

$$R_N = 1 - \frac{\alpha_0(Q^2)}{4\pi} \frac{1}{2b_0} \left[\gamma^{(1)N}_{NS} - 2 b_0 C^{(1)N}_{2,q} - \frac{b_1}{b_0} \gamma^{(0)N}_{NS} \left(1 + \log \log Q^2/\Lambda^2 \right) \right] \qquad (6.2)$$

$b_0 = -\frac{25}{3}$, $b_1 = -\frac{154}{3}$ and

$$\alpha_0(Q^2) = -\frac{4\pi}{b_0 \log Q^2/\Lambda^2}$$

This quantity gives the full second order modification of the Q dependence of the moments of the structure function F_2. In the time-like region the corresponding modification of the fragmentation function $\bar{F}_2$ is given by

$$\bar{R}_N = R_N + \frac{\alpha_0(Q^2)}{4\pi} \frac{1}{2b_0} \Delta K^{(1)N}_{2,NS} \tag{6.3}$$

where $\Delta K^{(1)N}_{2,NS}$ is given in eq. (5.23).

The numerical values of R_N and $\bar{R}_N$ as functions of N for $Q = 10$ and Λ 0.3 are plotted in fig. 8 which exhibits the large violation of the reciprocity relation. When the π^2 term is removed, that is when we subtract $\frac{\alpha}{4\pi} \frac{8}{3} \pi^2$ to $\bar{R}_N$, this violation is strongly reduced but remains appreciable at low N values.

This large violation of the GL reciprocity relation directly asks for the validity of the perturbative expansion of the different quantities we are looking for

$$A_N = A_N^{(0)} \left[1 + \frac{\alpha}{4\pi} A_N^{(1)} + \cdots \right] \tag{6.4}$$

With $\alpha \sim .3$, we require $A_N^{(1)}$ to be small as compared to~42 in order to be confident in the perturbative expansion. In fig. 9a are plotted $C^{(1)N}_{2,q}$, $\bar{C}^{(1)N}_{2,q}$ and $\bar{C}^{(1)N}_{2,q} - \frac{8}{3}\pi^2$, exhibiting the bad convergence of the coefficient function, especially in the time-like region. In contrast with this the perturbative expansion of anomalous dimension always has a very good convergence, with only a different low N behaviour between space and time-like cases. The behaviour of the coefficient function will be reflected in the expansion of the moments of the evolution kernel corresponding to eq. (2.14). Considering this expansion (6.4) as an expansion in the parameter α, we have an acceptable convergence in the space-like region (the $\log N$ gives a very slow increase) but a very bad one in the time-like region as seen in fig. 9c. Let us note that we are not looking for the second order effects in

the scaling violation of the experimental data given by a relation like eq. (6.1) but we are asking for the validity of the perturbative expansion of the evolution kernel.

Such an analysis can also be made in x space where we can compare first and second order terms :

$$Q\frac{\partial}{\partial Q}\overset{(-)}{F}(x,Q) = \overset{(-)}{K}(x,\alpha) \otimes \overset{(-)}{F}(x,Q)$$

$$= \frac{\alpha}{4\pi}\overset{(-)}{K}{}^{(0)}(x) \otimes F(x,Q) + \left(\frac{\alpha}{4\pi}\right)^2 \overset{(-)}{K}{}^{(1)}(x) \otimes F(x,Q) . \qquad (6.5)$$

In order to estimate the G.L. reciprocity violation, we apply the evolution kernel to the (experimental) structure function

$$x F_3(x,Q_0) = 2.24\ x^{0.42}(1-x)^{2.26} \qquad 0 \le x \le 1 \qquad (6.6)$$

and to its reciprocal fragmentation function

$$\frac{1}{x^2}\bar{F}_3(x,Q_0) = 2.24\ \frac{1}{x^{0.42}}\left(1-\frac{1}{x}\right)^{2.26} \qquad 1 \le x < \infty \qquad (6.7)$$

Fig. 10 shows such a comparison for $\alpha(Q_0) = 0.31$, a value which corresponds for $Q_0 = 3\,GeV$ to $\Lambda = 0{,}5$ with

$$\frac{\alpha(Q)}{4\pi} = -\frac{1}{b_0 \log Q^2/\Lambda^2} - \frac{b_1}{b_0^3}\frac{\log\log Q^2/\Lambda^2}{\log^2 Q^2/\Lambda^2} . \qquad (6.8)$$

In the space-like region the second order contribution does not change too much the first-order derivative, as expected from the momentum analysis. Such a change would even be smaller if we had compared the second-order contribution with a first-order kernel using the leading order $\alpha_0(Q)$.

In the time-like region the first-order derivative is the same as in the space-like case because the G.L. relation is conserved by convolution. But now the contribution of the second-order kernel is very important and it is as big as the first-order one for most values of X . This fact again was expected

from the moment analysis where we have shown that the moment values are between 30 and 40.

Such a result directly asks for the validity of the perturbative expansion in the time-like region (if $\Lambda_{T.L} = \Lambda_{S.L}$) and we will discuss this problem in our conclusions. Nevertheless we can compare the evolution of the structure and fragmentation functions (6.6) and (6.7) when next-to-leading log corrections are included. Fig. 11 shows such an evolution from $Q_0 = 3$ GeV up to $Q = 30$ GeV with $\Lambda = 0.5$ and $\alpha(Q)$ given by (6.8). One easily realizes from figs. 11a and 11b that the fragmentation function evolves more rapidly than the structure function for $X > 0.1$. More precisely, $\frac{1}{X^2}\,\overline{F}_3(X, 10\text{ Gev})$ is very similar, when plotted in $\frac{1}{X}$, to $X F_3(X, 30\text{ GeV})$ for $X > 0.2$.

6.2 The singlet case

The analysis of the reciprocity relation in the case of singlet kernel is not so easy because of the mixing between quarks and gluons. As we want to look for the second order contribution to scaling violation of physical structure function, the diagonalization of the first-order kernel is not adequate. So that we directly consider the evolution in X space given by eq. (2.21)

$$Q\frac{\partial}{\partial Q}\overset{(-)}{F}_2(x,Q) = \left[\frac{\alpha}{4\pi}\overset{(-)}{K}{}^{(0)}_{22}(x) + \left(\frac{\alpha}{4\pi}\right)^2 \overset{(-)}{K}{}^{(1)}_{22}(x)\right] \otimes \overset{(-)}{F}_2(x,Q) + \left[\frac{\alpha}{4\pi}\overset{(-)}{K}{}^{(0)}_{2g}(x) + \left(\frac{\alpha}{4\pi}\right)^2 \overset{(-)}{K}{}^{(1)}_{2g}(x)\right] \otimes \overset{(-)}{\lambda}\cdot\overset{(-)}{G}(x,Q) \tag{6.9a}$$

$$Q\frac{\partial}{\partial Q}\overset{(-)}{\lambda}\cdot\overset{(-)}{G}(x,Q) = \left[\frac{\alpha}{4\pi}\overset{(-)}{K}{}^{(0)}_{g2}(x) + \left(\frac{\alpha}{4\pi}\right)^2 \overset{(-)}{K}{}^{(1)}_{g2}(x)\right] \otimes \overset{(-)}{F}_2(x,Q) + \left[\frac{\alpha}{4\pi}\overset{(-)}{K}{}^{(0)}_{gg}(x) + \left(\frac{\alpha}{4\pi}\right)^2 \overset{(-)}{K}{}^{(1)}_{gg}(x)\right] \otimes \overset{(-)}{\lambda}\cdot\overset{(-)}{G}(x,Q) \tag{6.9b}$$

As an example we consider the initial conditions

$$x F_2(x,Q_0) = 2.24\, x^{0.42}(1-x)^{2.26} + 0.5\,(1-x)^{6.99} \tag{6.10a}$$

$$\frac{1}{x^2}\bar{F}_2(x,Q_0) = 2.24\ \frac{1}{x^{0.42}}\left(1-\frac{1}{x}\right)^{2.26} + 0.5\left(1-\frac{1}{x}\right)^{6.99} \tag{6.10b}$$

$$x\,G(x,Q_0) = 1.25\left[\,0.5\,x^{0.42}(1-x)^{2.26} + 2.24\,(1-x)^{6.99}\,\right] \tag{6.10c}$$

$$\frac{1}{x^2}\bar{G}(x,Q_0) = 1.25\left[\,0.5\,\frac{1}{x^{0.42}}\left(1-\frac{1}{x}\right)^{2.26} + 2.24\left(1-\frac{1}{x}\right)^{6.99}\right] \tag{6.10d}$$

The expressions (6.10a) and (6.10c) have been chosen because they seem close to the experimental data of deep inelastic scattering around $Q_0 = 3$ GeV, from which we deduce (6.10b) and (6.10c) by the reciprocity relation.

The first-order matrix elements of the evolution kernel (6.9) are nothing else than the usual $P^{(0)}_{qq}$, $P^{(0)}_{qg}$, $P^{(0)}_{gq}$ and $P^{(0)}_{gg}$, and satisfy

$$\bar{K}^{(0)}_{22}(x) = x\,K^{(0)}_{22}\left(\frac{1}{x}\right) \tag{6.11a}$$

$$\bar{K}^{(0)}_{2g}(x) = -\frac{2T_R}{C_F}\,x\,K^{(0)}_{g2}\left(\frac{1}{x}\right) \tag{6.11b}$$

$$\bar{K}^{(0)}_{g2}(x) = -\frac{C_F}{2T_R}\,x\,K^{(0)}_{2g}\left(\frac{1}{x}\right) \tag{6.11c}$$

$$K^{(0)}_{gg}(x) = x\,K^{(0)}_{gg}\left(\frac{1}{x}\right) \tag{6.11d}$$

In our normalization $\bar{\lambda}/\lambda = -C_F/2T_R$, these relations represent the Gribov-Lipatov relation (6.2) in the LLA. These relations do not imply the "naive" G.L relation for $F_{2,S}$, $\bar{F}_{2,S}$ and G, $\bar{G}$ because of the transposition of the evolution kernel in the time-like region. We thus obtain a violation of the G.L reciprocity relation in the first-order evolution of $\bar{F}_2$ and $\bar{G}$. Let us point out that the fact that the product of the non-diagonal matrix elements satisfies the G.L. relation implies that the diagonalized distribution evolution

would satisfy such a relation but that the matrix which expresses the F_2 and G in terms of these diagonalized distributions does not satisfy this relation. In fig. 12 one can see that the effect is important when comparing the solid to the dot dashed derivative, in particular for F_2 near $X = 0$.

In what concerns the second-order effect (dashed (dotted) for space (time) like region) we reach the same conclusion as in the non-singlet case, namely that the second-order effect is indeed a small perturbation of the first-order effect for the space-like case while it is comparable to the first-order effect in the time-like case. In particular for $\overline{F_2}$ in the region $\frac{1}{x} > 0.2$ we have a situation very similar to the non-singlet case because the main contribution comes from $\overline{K}^{(1)}_{22}$ which is essentially the same as $\overline{K}^{(1)}_{NS}$.

6.3 Conclusions

The main result of this work is that for the second-order anomalous dimensions and coefficient functions there is an analytic continuation relation when going from the time-like region to the space-like one. The first problem which arises is the validity of such a relation at higher orders. As in this continuation relation one has to take the real parts of the different quantities, there is no general way to resolve this problem. On the other hand the coupled evolution equations for F_2 and G contain terms of the form $e^{(1)} \otimes P^{(0)}$ which are the convolution of two real parts, and not the real part, of an analytic continuation. Such terms which represent the third order of the coefficient function may be an indication that the analytic continuation would not be true in higher orders. In this respect a calculation of this third order, which is not too complicated, would be very interesting.

The second problem which emerges is the validity of the perturbative expansion, namely with the occurrence of π^2 terms in the time-like region.

As such terms are related to the infrared behaviour, one could expect exponentiation of it. In fact such an exponentiation also concerns some other terms appearing both in the space and time-like regions. Some conjecture has been made for such an exponentiation [30] and here again calculation of the α^2 order term for the coefficient function would be a strong test for that. A clear understanding of this situation is important to clarify the possible next-to-leading log analysis of experimental data, in particular in e^+e^- annihilation where some data are appearing [31].

APPENDIX A

Here we give more details of the calculation of a cut vertex diagram in the space-like and time-like regions.

Consider the diagrams in fig. 7 we first calculate the subgraph which contains the cut vertex and has external momentum K_2 (fig. 6)

$$D_{6a}+D_{6b} = ig^2 C_F \mu^{2\varepsilon_1} \int \frac{d^{n_1}K_1}{(2\pi)^{n_1}} \left[\theta(p_1)\delta(p_1-x)\, abs - \right. \tag{A.1}$$

$$\left. - \delta(p_2-x) \right] \frac{\not{p}\, \not{K}_1\, \not{p}}{(K_1^2+i0)\left[(K_1-K_2)^2+i0\right]}$$

Here we use different dimensions for each loop momentum $n_i = 4-2\varepsilon_i$, $i=1,2$. The operation "abs" is used in order to determine the absorptive part with respect to the variable $(K_2-K_1)^2$. Relations (3.1) and (3.3) tell us that the integral (A.1) is

$$D_{6a}+D_{6b} = \mp \frac{2\alpha\cdot C_F}{4\pi} \not{p}\, p_2^{-1}\, \Gamma(\varepsilon_1) \left\{ \left(\frac{x}{p_2}\right)^{1-\varepsilon_1} \left|1-\frac{x}{p_2}\right|^{-1-\varepsilon_1} \cdot |K_2^2|^{-\varepsilon_1} \right.$$

$$\left. \mp (-K_2^2-i0)^{-\varepsilon_1}\, \delta\left(1-\frac{x}{p_2}\right) \int_0^1 dp_1\, p_1^{1-\varepsilon_1} (1-p_1)^{-\varepsilon_1-1} \right\} \tag{A.2}$$

where - and + correspond to the space and time-like cases, respectively.

We now perform the second loop momentum integration.

$$D_{7a}+D_{7b} = -i(\mp)\, \frac{2\alpha C_F}{4\pi} (4\pi\mu^2)^{\varepsilon_1} \Gamma(\varepsilon_1)\, g^2 \mu^{2\varepsilon_2} \cdot \frac{1}{4\Delta\cdot p}$$

$$\int \frac{d^{n_2}K_2}{(2\pi)^{n_2}} \operatorname{tr}\left[\not{p}\gamma^\mu \not{K}_2 \not{p} \not{K}_2 \gamma_\mu\right] \Delta\cdot K_2 \left\{ \left(\frac{x}{p_2}\right)^{1-\varepsilon_1} \left|1-\frac{x}{p_2}\right|^{-\varepsilon_1-1} \cdot \right.$$

$$\cdot |K_2^2|^{-\varepsilon_1} \mp \delta\left(1-\frac{x}{p_2}\right) (-K_2^2-i0)^{-\varepsilon_1} \cdot \int_0^1 dp_1\, p_1^{1-\varepsilon_1} (1-p_1)^{-\varepsilon_1-1} \Big\} \cdot$$

$$\cdot\, abs\, \frac{1}{\left[(p-K_2)^2+i0\right]} \tag{A.3}$$

The trace gives

$$\frac{1}{4\Delta\cdot p}\,tr\left[\not p\gamma^{\mu}\not k_2\not\Delta\not k_2\gamma_{\mu}\right]=-2(1-\varepsilon_2)\left(p^2\rho_2-(1-\rho_2)k_2^2-\rho_2(p-k_2)^2\right)$$

Once more if we apply relations (3.1) and (3.3) and the constraint $\rho_2\geq x$, we obtain

$$D_{\gamma a}+D_{\gamma b}=4C_F^2\left(\frac{\alpha}{4\pi}\right)^2\left|\frac{p^2}{4\pi\mu^2}\right|^{-\varepsilon_1-\varepsilon_2}(1-\varepsilon_2)\,\frac{\Gamma(\varepsilon_1+\varepsilon_2)}{\varepsilon_1}\cdot$$
$$\int_x^1\frac{d\rho_2}{\rho_2}\,\rho_2^{-\varepsilon_1-\varepsilon_2}\,|1-\rho_2|^{-\varepsilon_2}\left\{\left(\frac{x}{\rho_2}\right)^{1-\varepsilon_1}\left|1-\frac{x}{\rho_2}\right|^{-1-\varepsilon_1}\right.$$
$$\left.\pm\,e^{i\pi\varepsilon_1}\,\delta\left(1-\frac{x}{\rho_2}\right)\int_0^1 d\rho_1\,\rho_1^{1-\varepsilon_1}(1-\rho_1)^{-\varepsilon_1-1}\right\}\left[1-\rho_2+\frac{\varepsilon_1+\varepsilon_2}{1+\varepsilon_1}\right]. \tag{A.4}$$

The above expression is real in the space-like region $p^2<0$ and complex in the time-like region. It becomes real after inclusion of the conjugate diagrams (7c), (7d). In order to find the contribution to the anomalous dimensions $P(x)$ and $\bar{P}(x)$ we must subtract the counterterms (C.T.) and expand in ε. We kept two different ε in order to facilitate the extraction of the C.T. Indeed the only subdivergence is found directly from (A.4) looking at the ε_1 pole,

$$C.T.=-4C_F^2\left(\frac{\alpha}{4\pi}\right)^2\left|\frac{p^2}{4\pi\mu^2}\right|^{-\varepsilon_2}(1-\varepsilon_2)\,\frac{\Gamma(\varepsilon_2)}{\varepsilon_1}\cdot\int_x^1\frac{d\rho_2}{\rho_2}\,\rho_2^{-\varepsilon_2}\,|1-f_2|^{-\varepsilon_2}$$
$$*\left\{\frac{x}{\rho_2}\left|1-\frac{x}{\rho_2}\right|^{-1-\varepsilon_1}\pm\,\delta\left(1-\frac{x}{\rho_2}\right)\int_0^1 d\rho_1\,\rho_1\,(1-\rho_1)^{-\varepsilon_1-1}\right\}\left[1-\rho_2+\varepsilon_2\right] \tag{A.5}$$

Now we put $\varepsilon_1=\varepsilon_2=\varepsilon$ in (A.4) and in (A.5) and expand in ε. When we add (A.4) and (A.5) we observe that the pole term proportional to $\log(-p^2\cdot i0)$ disappears as it should. The remaining simple pole terms give the following

contribution to $P(x)$ and $\bar{P}(x)$, respectively

$$8C_F^2 \left[-6(1-x) - 2x \log x - 4(1-x) \log(1-x) + x \log^2 x \right. \\ \left. - (1-x) \log^2(1-x) - 4(1-x) S(1-x) \right] \tag{A.6}$$

and

$$-8C_F^2 \left[-6(1-x) - 2x \log x - 4(1-x) \log(x-1) + x \log^2 x \right. \\ \left. - (1-x) \log^2(x-1) - (1-x)\pi^2 - 4(1-x) S(1-x) \right] \tag{A.7}$$

where

$$S(x) = -\frac{\pi^2}{6} + \int_0^x d\xi \; \frac{\log(1-\xi)}{\xi}$$

We observe that to go from the time-like region (A.7) to the space-like one (A.6) we must take the following analytic continuation

$$P(x) = -\mathrm{Re}\, \bar{P}(x) \quad , \quad 0 < x < 1 \, .$$

Finally, we observe that taking moments in the space-like case we obtain the same answer as that of ref. [12] for the same diagram, as it is compactified in ref. [21].

APPENDIX B

In this appendix we present the general forms of the quantities needed for a next-to-leading log analysis. The coefficient functions and anomalous dimensions are expanded as

$$C_{i,2}^{N} = C_{i,2}^{(0)N} + \frac{\alpha}{4\pi} C_{i,2}^{(1)N} \tag{B.1a}$$

$$e_{i,2}(x) = e_{i,2}^{(0)}(x) + \frac{\alpha}{4\pi} e_{i,2}^{(1)}(x) \tag{B.1b}$$

$$\gamma_{NS}^{N} = \frac{\alpha}{4\pi} \gamma_{NS}^{(0)N} + \left(\frac{\alpha}{4\pi}\right)^2 \gamma_{NS}^{(1)N} \tag{B.2a}$$

$$P_{NS}(x) = \frac{\alpha}{4\pi} P_{NS}^{(0)}(x) + \left(\frac{\alpha}{4\pi}\right)^2 P_{NS}^{(1)}(x) \tag{B.2b}$$

$$\gamma_{rr'}^{N} = \frac{\alpha}{4\pi} \gamma_{rr'}^{(0)N} + \left(\frac{\alpha}{4\pi}\right)^2 \gamma_{rr'}^{(1)N} \tag{B.3a}$$

$$P_{rr'}(x) = \frac{\alpha}{4\pi} P_{rr'}^{(0)}(x) + \left(\frac{\alpha}{4\pi}\right)^2 P_{rr'}^{(1)}(x) \tag{B.3b}$$

where $r,r'=q,g$ and $i=2,3,L$ ($e_1=e_2+2e_L$) and with similar expansion in the time-like region.

We start with the moment space expressions in the space-like region [9,10,23,24]

$$C_{2,g}^{(0)N} = 1 \tag{B.4}$$

$$C_{3,g}^{(0)N} = 1 \tag{B.5}$$

$$C_{L,g}^{(0)N} = 0 \tag{B.6}$$

$$C_{2,g}^{(1)N} = C_F\left[2S_1^2(N) - 2S_2(N) + 3S_1(N) - \frac{2S_1(N)}{N(N+1)} + \frac{3}{N} + \frac{4}{N+1} + \frac{2}{N^2} - 9\right] \tag{B.7}$$

$$C_{3,g}^{(1)N} = C_F\left[2S_1^2(N) - 2S_2(N) + 3S_1(N) - \frac{2S_1(N)}{N(N+1)} + \frac{1}{N} + \frac{2}{N+1} + \frac{2}{N^2} - 9\right] \tag{B.8}$$

$$C_{L,q}^{(1)N} = C_F \frac{2}{N+1} \tag{B.9}$$

$$C_{2,g}^{(0)N} = 0 \tag{B.10}$$

$$C_{L,g}^{(0)N} = 0 \tag{B.11}$$

$$C_{2,g}^{(1)N} = -4 T_R \left[S_1(N) \frac{N^2+N+2}{N(N+1)(N+2)} + \frac{1}{N} - \frac{1}{N^2} - \frac{6}{N+1} + \frac{6}{N+2} \right] \tag{B.12}$$

$$C_{L,g}^{(1)N} = +8 T_R \frac{1}{(N+1)(N+2)} \tag{B.13}$$

$$\gamma_{NS}^{(0)N} = \gamma_{qq}^{(0)N} = 2 C_F \left[4 S_1(N) - 3 - \frac{2}{N(N+1)} \right] \tag{B.14}$$

$$\gamma_{qg}^{(0)N} = -8 T_R \frac{N^2+N+2}{N(N+1)(N+2)} \tag{B.15}$$

$$\gamma_{gq}^{(0)N} = -4 C_F \frac{N^2+N+2}{(N-1)N(N+1)} \tag{B.16}$$

$$\gamma_{gg}^{(0)N} = 2 C_A \left[4 S_1(N) - \frac{11}{3} - \frac{4}{N(N-1)} - \frac{4}{(N+1)(N+2)} \right] + \frac{8}{3} T_R \tag{B.17}$$

$$\begin{aligned}
\gamma_{NS}^{(1)N}(\eta) = C_F^2 \Bigg[& \frac{16 S_1(N)(2N+1)}{N^2(N+1)^2} + 16 \left(2 S_1(N) - \frac{1}{N(N+1)} \right) \left(S_2(N) - S_2'\left(\frac{N}{2}\right) \right) \\
& + 24 S_2(N) + 64 \widetilde{S}(N) - 8 S_3'\left(\frac{N}{2}\right) - 3 \\
& - 8 \frac{3N^3+N^2-1}{N^3(N+1)^3} - 16 \eta \frac{2N^2+2N+1}{N^3(N+1)^3} \Bigg] \\
+ C_A C_F \Bigg[& \frac{536}{9} S_1(N) - 8 \left(2 S_1(N) - \frac{1}{N(N+1)} \right) \left(2 S_2(N) - S_2'\left(\frac{N}{2}\right) \right) \\
& - \frac{88}{3} S_2(N) - 32 \widetilde{S}(N) + 4 S_3'\left(\frac{N}{2}\right) - \frac{17}{3} \\
& - \frac{4}{9} \frac{151N^4+236N^3+88N^2+3N+18}{N^3(N+1)^3} + 8 \eta \frac{2N^2+2N+1}{N^3(N+1)^3} \Bigg] \\
+ C_F T_R \Bigg[& -\frac{160}{9} S_1(N) + \frac{32}{3} S_2(N) + \frac{4}{3} + \frac{16}{9} \frac{11N^2+5N-3}{N^2(N+1)^2} \Bigg]
\end{aligned} \tag{B.18}$$

$$\gamma_{qq}^{(1)N} = \gamma_{NS}^{(1)N}(1) - 16 C_F T_R \frac{5N^5+32N^4+49N^3+38N^2+28N+8}{(N-1)N^3(N+1)^3(N+2)^2} \qquad (B.19)$$

$$\gamma_{qg}^{(1)N} = -8 C_A T_R \left[\left(-2S_1^2(N) + 2S_2(N) - 2S_2'\left(\frac{N}{2}\right)\right) \frac{N^2+N+2}{N(N+1)(N+2)} + \frac{8S_1(N)(2N+3)}{(N+1)^2(N+2)^2} \right.$$
$$\left. + 2 \frac{N^9+6N^8+15N^7+25N^6+36N^5+85N^4+128N^3+104N^2+64N+16}{(N-1)N^3(N+1)^3(N+2)^3} \right]$$
$$- 8 C_F T_R \left[\left(2S_1^2(N) - 2S_2(N) + 5\right) \frac{N^2+N+2}{N(N+1)(N+2)} - \frac{4S_1(N)}{N^2} \right. \qquad (B.20)$$
$$\left. + \frac{11N^4+26N^3+15N^2+8N+4}{N^3(N+1)^3(N+2)} \right]$$

$$\gamma_{gq}^{(1)N} = -4 C_F^2 \left[\left(-2S_1^2(N) + 10S_1(N) - 2S_2(N)\right) \frac{N^2+N+2}{(N-1)N(N+1)} - \frac{4S_1(N)}{(N+1)^2} \right.$$
$$\left. - \frac{12N^6+30N^5+43N^4+28N^3-N^2-12N-4}{(N-1)N^3(N+1)^3} \right]$$
$$- 8 C_F C_A \left[\left(S_1^2(N) + S_2(N) - S_2'\left(\frac{N}{2}\right)\right) \frac{N^2+N+2}{(N-1)N(N+1)} - S_1(N) \frac{17N^4+41N^2-22N-12}{3(N-1)^2N^2(N+1)} \right. \qquad (B.21)$$
$$\left. + \frac{109N^9+621N^8+1400N^7+1678N^6+695N^5-1031N^4-1304N^3-152N^2+432N+144}{9(N-1)^2N^3(N+1)^3(N+2)^2} \right]$$
$$- \frac{32}{3} C_F T_R \left[\left(S_1(N) - \frac{8}{3}\right) \frac{N^2+N+2}{(N-1)N(N+1)} + \frac{1}{(N+1)^2} \right]$$

$$\gamma_{gg}^{(1)N} = C_A T_R \left[-\frac{160}{9} S_1(N) + \frac{32}{3} + \frac{16}{9} \frac{38N^4+76N^3+94N^2+56N+12}{(N-1)N^2(N+1)^2(N+2)} \right]$$
$$+ C_F T_R \left[8 + 16 \frac{2N^6+4N^5+N^4-10N^3-5N^2-4N-4}{(N-1)N^3(N+1)^3(N+2)} \right]$$
$$+ C_A^2 \left[\frac{536}{9} S_1(N) + 64 S_1(N) \frac{2N^5+5N^4+8N^3+7N^2-2N-2}{(N-1)^2N^2(N+1)^2(N+2)^2} - \frac{64}{3} \right. \qquad (B.22)$$
$$+ 32 S_2'\left(\frac{N}{2}\right) \frac{N^2+N+1}{(N-1)N(N+1)(N+2)} - 16 S_1(N) S_2'\left(\frac{N}{2}\right) + 32 \tilde{S}(N) - 4 S_3'\left(\frac{N}{2}\right)$$
$$\left. - \frac{4}{9} \frac{457N^9+2742N^8+6040N^7+6098N^6+1567N^5-2344N^4-1632N^3+560N^2+1488N+576}{(N-1)^2N^3(N+1)^3(N+2)^3} \right]$$

where $\eta = \pm 1$ for continuation of even or odd momenta (for singlet anomalous dimension we only use $\eta = 1$), and

$$S_\ell(N) = \sum_{j=1}^{N} \frac{1}{j^\ell} \tag{B.23}$$

$$\tilde{S}(N) = \sum_{j=1}^{N} (-)^j \frac{S_1(j)}{j^2} \tag{B.24}$$

$$S'_\ell\left(\frac{N}{2}\right) = S_\ell\left(\text{integer part of } \frac{N}{2}\right) = 2^{\ell-1} \sum_{j=1}^{N} \frac{1+(-)^j}{j^\ell} \tag{B.25}$$

By inverse Mellin transform one obtains the corresponding functions of x which by analytic continuation (eq. 4.6) will also represent the time-like expression. These functions are listed below according to their convolution with any trial function $\phi(x)$ as

$$K(x) \otimes \phi(x) = K^{[1]}(x)\,\phi(x) + \int_x^1 K^{[2]}(y)\,\phi(x/y)\,\frac{dy}{y} + \int_x^1 K^{[3]}(y)\left(\frac{1}{y}\,\phi(x/y) - \phi(x)\right) dy \tag{B.26}$$

So that the functions $\bar{K}^{[i]}(x)$ are the time-like expressions for $x > 1$ and $K^{[i]}(x)$ the space-like ones which are the real parts of the analytic continuation to $x < 1$.

$$\overset{(-)}{e}{}^{(0)[1]}_{2,q}(x) = \delta(1-x) \tag{B.27a}$$

$$\overset{(-)}{e}{}^{(0)[i]}_{2,q}(x) = 0 \qquad , i = 2, 3 \tag{B.27b}$$

$$\overset{(-)}{e}{}^{(0)[1]}_{3,q}(x) = \delta(1-x) \tag{B.28a}$$

$$\overset{(-)}{e}{}^{(0)[i]}_{3,q}(x) = 0 \qquad , i = 2, 3 \tag{B.28b}$$

$$\bar{e}^{(0)[i]}_{L,q}(x) = 0 \qquad , i = 1,2,3 \tag{B.29}$$

$$\bar{e}^{(1)[1]}_{2,q}(x) = C_F\left[-9 - \frac{2}{3}\pi^2 + 2\,\mathrm{Re}\left(\log^2(x-1) + \pi^2\right) - 3\log|1-x|\right] \tag{B.30a}$$

$$\bar{e}^{(1)[2]}_{2,q}(x) = C_F\left[6 + 4x - 2\,\frac{1+x^2}{1-x}\log x - 2(1+x)\log|1-x|\right] \tag{B.30b}$$

$$\bar{e}^{(1)[3]}_{2,q}(x) = C_F\,\frac{-3 + 4\log|1-x|}{1-x} \tag{B.30c}$$

$$\bar{e}^{(1)[1]}_{3,q}(x) = \bar{e}^{(1)[1]}_{2,q}(x) \tag{B.31a}$$

$$\bar{e}^{(1)[2]}_{3,q}(x) = C_F\left[4 + 2x - 2\,\frac{1+x^2}{1-x}\log x - 2(1+x)\log|1-x|\right] \tag{B.31b}$$

$$\bar{e}^{(1)[3]}_{3,q}(x) = \bar{e}^{(1)[3]}_{2,q}(x) \tag{B.31c}$$

$$\bar{e}^{(1)[i]}_{L,q}(x) = 0 \qquad , i = 1,3 \tag{B.32a}$$

$$\bar{e}^{(1)[2]}_{L,q}(x) = 2C_F\,x \tag{B.32b}$$

$$\bar{e}^{(0)[i]}_{2,g}(x) = 0 \qquad , i = 1,2,3 \tag{B.33}$$

$$\bar{e}^{(0)[i]}_{L,g}(x) = 0 \qquad , i = 1,2,3 \tag{B.34}$$

$$\bar{e}^{(1)[i]}_{2,g}(x) = 0 \qquad , i = 1,3 \tag{B.35a}$$

$$\bar{e}^{(1)[2]}_{2,g}(x) = -4T_R\left[-3 + (1-2x+2x^2)\left(4 - \log\frac{|1-x|}{x}\right)\right] \tag{B.35b}$$

$$\tilde{e}^{(1)[i]}_{q,g}(x) = 0 \qquad , i = 1,3 \tag{B.36a}$$

$$\tilde{e}^{(1)[2]}_{q,g}(x) = +8T_R\, x(1-x) \tag{B.36b}$$

$$\tilde{P}^{(0)[1]}_{NS}(x) = 2C_F\,(3+4\log|1-x|) \tag{B.37a}$$

$$\tilde{P}^{(0)[2]}_{NS}(x) = -4C_F\,(1+x) \tag{B.37b}$$

$$\tilde{P}^{(0)[3]}_{NS}(x) = \frac{8C_F}{1-x} \tag{B.37c}$$

$$\tilde{P}^{(0)[i]}_{qq}(x) = \tilde{P}^{(0)[i]}_{NS}(x) \qquad , i = 1,2,3 \tag{B.38}$$

$$\tilde{P}^{(0)[i]}_{qg}(x) = 0 \qquad , i = 1,3 \tag{B.39a}$$

$$\tilde{P}^{(0)[2]}_{qg}(x) = 8T_R\,(1-2x+2x^2) \tag{B.39b}$$

$$\tilde{P}^{(0)[i]}_{gq}(x) = 0 \qquad , i = 1,3 \tag{B.40a}$$

$$\tilde{P}^{(0)[2]}_{gq}(x) = 4C_F\left(\frac{2}{x}-2+x\right) \tag{B.40b}$$

$$\tilde{P}^{(0)[1]}_{gg}(x) = -\frac{8}{3}T_R + 2C_A\left(\frac{11}{3}+4\log|1-x|\right) \tag{B.41a}$$

$$\tilde{P}^{(0)[2]}_{gg}(x) = 8C_A\left(\frac{1}{x}-2+x-x^2\right) \tag{B.41b}$$

$$\tilde{P}^{(0)[3]}_{gg}(x) = \frac{8C_A}{1-x} \tag{B.41c}$$

$$\tilde{P}_{NS}^{(1)[1]}(x) = -\frac{16}{9} C_F T_R \left(\frac{3}{4} + \pi^2 + 10 \log|1-x| \right)$$
$$+ C_F^2 \left(3 - 4\pi^2 + 48 Li_3(1) \right)$$
$$+ C_A C_F \left[\frac{17}{3} + \frac{44}{9}\pi^2 - 24 Li_3(1) + \left(\frac{536}{9} - \frac{8}{3}\pi^2 \right) \log(1-x) \right] \qquad \text{(B.42a)}$$

$$\tilde{P}_{NS,\eta}^{(1)[2]}(x) = \frac{16}{9} C_F T_R \left[-1 + 11x - 3\frac{1+x^2}{1-x} \log x \right]$$
$$+ C_F^2 \left[-40(1-x) - 8\left(2x + \frac{3}{1-x}\right)\log x - 4(1+x)\log^2 x \right.$$
$$\left. - 16\frac{1+x^2}{1-x}\log x \log|1-x| + 16\eta\left(2(1-x) + (1+x)\log x + \frac{1+x^2}{1+x} F(x)\right) \right] \qquad \text{(B.42b)}$$
$$+ C_A C_F \left[\frac{4}{9}(53 - 187x) - \frac{4}{3}\left(5 + 5x - \frac{22}{1-x}\right)\log x + 4\frac{1+x^2}{1-x}\log^2 x \right.$$
$$\left. + 4(1+x)\frac{\pi^2}{3} - 8\eta\left(2(1-x) + (1+x)\log x + \frac{1+x^2}{1+x} F(x)\right) \right]$$

$$\tilde{P}_{NS}^{(1)[3]}(x) = \left[-\frac{160}{9} C_F T_R + \left(\frac{536}{9} - \frac{8}{3}\pi^2 \right) C_A C_F \right] \frac{1}{1-x} \qquad \text{(B.42c)}$$

$$\tilde{P}_{qq}^{(1)[i]}(x) = \tilde{P}_{NS}^{(1)[i]}(x) \qquad , i = 1, 3 \qquad \text{(B.43a)}$$

$$\tilde{P}_{qq}^{(1)[2]}(x) = \frac{16}{9} C_F T_R \left[\frac{20}{\lambda} - 19 + 65x - 56x^2 + 6\left(2 + 8x + 4x^2 - \frac{1}{1-x}\right)\log x \right.$$
$$\left. - 9(1+x)\log^2 x \right]$$
$$+ C_F^2 \left[-8(1-x) + 8\left(2 - \frac{3}{1-x}\right)\log x - 4(1+x)\log^2 x \right. \qquad \text{(B.43b)}$$
$$\left. - 16\frac{1+x^2}{1-x}\log x \log|1-x| + 16\frac{1+x^2}{1+x} F(x) \right]$$
$$+ C_A C_F \left[\frac{4}{9}(17 - 151x) + 4\frac{1+x^2}{1-x}\log x \left(\frac{11}{3} + \log x \right) \right.$$
$$\left. + 4(1+x)\frac{\pi^2}{3} - 8\frac{1+x^2}{1+x} F(x) \right]$$

$$\overset{(-)}{P}{}^{(1)[i]}_{qg}(x) = 0 \quad , \qquad i = 1, 3 \tag{B.44a}$$

$$\begin{aligned}
\overset{(-)}{P}{}^{(1)[2]}_{qg}(x) = -8 C_F T_R \Big[& -14 + 29x - 20x^2 - (3 - 4x + 8x^2) \log x \\
& - (1 - 2x + 4x^2) \log^2 x \; - 8x(1-x) \log|1-x| \\
& + 2(1 - 2x + 2x^2) \left(2 \log x \log|1-x| + \frac{\pi^2}{3} - \mathrm{Re}\left(\log^2(x-1) + \pi^2 \right) \right) \Big] \\
-8 C_A T_R \Big[& -\frac{2}{9}\left(\frac{20}{x} - 18 + 225x - 218x^2 \right) - \frac{2}{3}(3 + 24x + 44x^2) \log x \\
& + (3 + 6x + 2x^2) \log^2 x \; + 8x(1-x) \log|1-x| \\
& + (1 - 2x + 2x^2) \left(2\,\mathrm{Re}\left(\log^2(x-1) + \pi^2 \right) - \frac{\pi^2}{3} \right) \\
& - 2(1 + 2x + 2x^2) \, F(x) \Big]
\end{aligned} \tag{B.44b}$$

$$\overset{(-)}{P}{}^{(1)[i]}_{gq}(x) = 0 \quad , \qquad i = 1, 3 \tag{B.45a}$$

$$\begin{aligned}
\overset{(-)}{P}{}^{(1)[2]}_{gq}(x) = & -\frac{32}{3} C_F T_R \left[\frac{2}{3}\left(\frac{5}{x} - 5 + 4x \right) + \left(\frac{2}{x} - 2 + x \right) \log|1-x| \right] \\
& -4 C_F^2 \Big[5 + 7x - (4 + 7x) \log x \; + 2\left(\frac{6}{x} - 6 + 5x \right) \log|1-x| \\
& + (2 - x) \log^2 x \; + 2\left(\frac{2}{x} - 2 + x \right) \mathrm{Re}\left(\log^2(x-1) + \pi^2 \right) \Big] \\
& -8 C_A C_F \Big[-\frac{1}{9}\left(\frac{9}{x} + 19 + 37x + 44x^2 \right) + \frac{1}{3}(36 + 15x + 8x^2) \log x \\
& - \frac{1}{2}\left(\frac{2}{x} + 6 + 3x \right) \log^2 x \; - \frac{1}{3}\left(\frac{22}{x} - 22 + 17x \right) \log|1-x| \\
& + \left(\frac{2}{x} - 2 + x \right) \left(2 \log x \log|1-x| + \frac{\pi^2}{6} - \mathrm{Re}\left(\log^2(x-1) + \pi^2 \right) \right) \\
& + \left(\frac{2}{x} + 2 + x \right) F(x) \Big]
\end{aligned} \tag{B.45b}$$

$$\tilde{P}_{gg}^{(1)\,[1]}(x) = -8C_F T_R - \frac{32}{3} C_A T_R \left(1 + \frac{5}{3}\log|1-x|\right) + \frac{8}{3} C_A^2 \left[8 + 9\zeta_3(1) + \left(\frac{67}{3} - \pi^2\right)\log|1-x|\right] \tag{B.46a}$$

$$\tilde{P}_{gg}^{(1)\,[2]}(x) = 8C_F T_R \left[\frac{4}{3}\left(\frac{1}{x} - 12 + 6x + 5x^2\right) - 2(3+5x)\log x - 2(1+x)\log^2 x\right] + \frac{32}{3} C_A T_R \left[-\frac{1}{6}\left(\frac{23}{x} - 29 + 19x - 23x^2\right) - (1+x)\log x\right] + 8C_A^2 \left[-\frac{1}{18}(25+109x) - \frac{1}{3}(25 - 11x + 44x^2)\log x + 4(1+x)\log^2 x - \frac{\pi^2}{3}\left(\frac{1}{x} - 2 + x - x^2\right) + \left(\frac{1}{x} - 2 + x - x^2 + \frac{1}{1-x}\right)\log x\left(\log x - 4\log|1-x|\right) - 2\left(\frac{1}{x} + 2 + x + x^2 - \frac{1}{1+x}\right) F(x)\right] \tag{B.46b}$$

$$\tilde{P}_{gg}^{(1)\,[3]}(x) = \left[-\frac{160}{9} C_F T_R + \frac{8}{3} C_A^2 \left(\frac{67}{3} - \pi^2\right)\right] \frac{1}{1-x} \tag{B.46c}$$

In formulae (B.42) to (B.46) we have defined

$$F(x) = \frac{1}{2}\log^2 x - 2\log x \log(1+x) - 2\,\mathrm{Li}_2(-x) - \frac{\pi^2}{6} \tag{B.47}$$

$$\mathrm{Li}_k(x) = \sum_n \frac{x^n}{n^k} \quad , \text{for } |x| \le 1 \tag{B.48}$$

$$= \int_0^x \frac{\mathrm{Li}_{k-1}(z)}{z}\, dz \quad , \quad \mathrm{Li}_1(z) = -\log(1-z) \tag{B.49}$$

So that $Li_3(1)=1.20205690$.

C_A, T_R and C_F are the usual quadratic Casimir operators.

In order to obtain the coupled evolution kernel for F_2 and D_g in eq.(2.21) we have to calculate the convolutions which appear in th expansion of eq.(2.22). Using

$$\bar{e}_{2q}^{-1}(x) = \delta(1-x) - \frac{\alpha}{4\pi}\,\bar{e}_{2q}^{(1)}(x) \tag{B.50}$$

this expansion up to second order reads as

$$K_{22} = \left(\frac{\alpha}{4\pi}\right) P_{qq}^{(0)} + \left(\frac{\alpha}{4\pi}\right)^2 \left[2b_0 e_{2q}^{(1)} + P_{qq}^{(1)} + e_{2g}^{(1)} \otimes P_{gq}^{(0)} \right] \tag{B.51a}$$

$$K_{2g} = \left(\frac{\alpha}{4\pi}\right) P_{qg}^{(0)} + \left(\frac{\alpha}{4\pi}\right)^2 \left[2b_0 e_{2g}^{(1)} + P_{qg}^{(1)} + e_{2q}^{(1)} \otimes P_{qg}^{(0)} + e_{2g}^{(1)} \otimes \left(P_{gg}^{(0)} - P_{qq}^{(0)} \right) \right] \tag{B.51b}$$

$$K_{g2} = \left(\frac{\alpha}{4\pi}\right) P_{gq}^{(0)} + \left(\frac{\alpha}{4\pi}\right)^2 \left[P_{gq}^{(1)} - e_{2q}^{(1)} \otimes P_{gq}^{(0)} \right] \tag{B.51c}$$

$$K_{gg} = \left(\frac{\alpha}{4\pi}\right) P_{gg}^{(0)} + \left(\frac{\alpha}{4\pi}\right)^2 \left[P_{gg}^{(1)} - e_{2g}^{(1)} \otimes P_{gq}^{(0)} \right] \tag{B.51d}$$

The convolution $e \otimes P$ can be obtained by inverse Mellin transform of the product of the moments in the space-like region. As for the corresponding results in the time-like region we have to take into account that the objects e and P in eq. (B.51) are real parts of the analytic continuation in such a way that $e_{2g} \otimes (P_{gg}-P_{qq})$ is no more the real part of the analytic continuation of $\bar{e}_{2g} \otimes (\bar{P}_{gg}-\bar{P}_{qq})$. The resulting convolutions are functions (they contribute only to $K^{[2]}$) and read

$$\overset{(-)(1)}{e}_{2,g} \otimes \overset{(-)(0)}{P}_{gq} = -\frac{16}{3} C_F T_R \Big[-2\left(\frac{1}{x} - 10 + x + 8x^2\right) + (3 + 15x - x^2)\log x$$
$$+ 6(1+x)\left(\mathrm{Re}\, Li_2(x) - \frac{\pi^2}{6}\log^2 x\right)$$
$$- \left(\frac{4}{x} + 3 - 3x - 4x^2\right)\log|1-x| \Big] \qquad \text{(B.52)}$$

$$\overset{(-)(1)}{e}_{2,q} \otimes P_{qg}^{(0)} = 8 C_F T_R \Big[-5 + 12x - 16x^2 - (1 + 8x - 12x^2)\log x$$
$$+ (1 - 2x + 4x^2)\log^2 x - (7 - 16x + 12x^2)\log|1-x|$$
$$- 2(1-2x)\left(\mathrm{Re}\, Li_2(x) - \frac{\pi^2}{6}\right) \qquad \text{(B.53)}$$
$$- 2(1 - 2x + 2x^2)\left(2\log x \log|1-x| + \frac{\pi^2}{3} - \mathrm{Re}\left(\log^2(x-1) + \pi^2\right)\right) \Big]$$

$$\overset{(-)(1)}{e}_{2g} \otimes \left(\overset{(-)(0)}{P}_{gg} - \overset{(-)(0)}{P}_{qq}\right) = -\frac{32}{3} T_R^2 \left[-1 + 8x - 8x^2 + (1 - 2x + 2x^2)\log\frac{|1-x|}{x} \right]$$
$$+ \frac{8}{3} C_A T_R \Big[-2\left(\frac{2}{x} - 27 - 77x + \frac{193}{2}x^2\right) + (17 + 170x - 40x^2)\log x$$
$$- \left(\frac{8}{x} + 5 + 122x - 136x^2\right)\log|1-x|$$
$$+ 12(1+4x)\left(\mathrm{Re}\, Li_2(x) - \frac{\pi^2}{6} + \frac{1}{2}\log^2 x\right) \Big] \qquad \text{(B.54)}$$
$$+ 8 C_F T_R \Big[-7 + 10x - (1 - 16x + 32x^2)\log x - (1 - 2x + 4x^2)\log^2 x$$
$$+ (5 - 36x + 32x^2)\log|1-x| - 2(1 - 2x + 4x^2)\left(\mathrm{Re}\, Li_2(x) - \frac{\pi^2}{6}\right) \Big]$$
$$- 32(C_A - C_F) T_R (1 - 2x + 2x^2)\left[\log x \log|1-x| + \frac{\pi^2}{6} - \log^2|1-x|\right]$$

$$\overset{(-)}{e}{}^{(1)}_{2q} \otimes P^{(0)}_{qq} = 4C_F^2 \Big[-3(1+2x) + 3x \log x - (2-x)\log^2 x - \Big(\frac{12}{x} - 16 + 7x\Big)\log|1-x| - 2\Big(\frac{4}{x} - 2 + x\Big)\Big(\mathrm{Re}\,Li_2(x) - \frac{\pi^2}{6}\Big) - 2\Big(\frac{2}{x} - 2 + x\Big)\Big(2\log x \log|1-x| + \frac{\pi^2}{3} - \mathrm{Re}\big(\log^2(x-1) + \pi^2\big)\Big)\Big] \tag{B.55}$$

Let us note that in (B.54) the last term $\log^2|1-x|$ does not appear in the analytic continuation combination $\mathrm{Re}(\log^2(x-1)+\pi^2)$. Thus this term violates the analytic continuation for the evolution kernel of singlet distribution.

Using the substitutions given in eq. (5.21), one obtains the expressions for the time-like moments , some of which we give below

$$\bar{C}^{(1)\,N+1}_{2,q} = C_F\Big[2S_1^2(N) + 2S_2(N) + 3S_1(N) - \frac{2S_1(N)}{N(N+1)} - \frac{4}{N} - \frac{3}{N+1} + \frac{2}{(N+1)^2} - 9 + \frac{4}{3}\pi^2\Big] \tag{B.56}$$

$$\bar{C}^{(1)\,N+1}_{2,g} = -4T_R\Big[-S_1(N)\frac{N^2+N+2}{(N-1)N(N+1)} - \frac{1}{N+1} - \frac{1}{(N+1)^2} + \frac{6}{N} - \frac{6}{N-1}\Big] \tag{B.57}$$

$$\begin{aligned}\bar{\gamma}^{(1)\,N+1}_{NS}(\eta) = C_F^2\Big[&-\frac{16S_1(N)(2N+1)}{N^2(N+1)^2} + 16\Big(2S_1(N) - \frac{1}{N(N+1)}\Big)\Big(3S_2(N) - S_2'\Big(\frac{N}{2}\Big) - \frac{\pi^2}{3}\Big) \\ &- 24S_2(N) + 64\tilde{S}(N) - 8S_3'\Big(\frac{N}{2}\Big) - 3 + 8\pi^2 \\ &+ 8\,\frac{3N^3+8N^2+7N+3}{N^3(N+1)^3} - 16\eta\,\frac{2N^2+2N+1}{N^3(N+1)^3}\Big] \\ + C_A C_F\Big[&\frac{536}{9}S_1(N) - 8\Big(2S_1(N) - \frac{1}{N(N+1)}\Big)\Big(2S_2(N) - S_2'\Big(\frac{N}{2}\Big)\Big) \\ &+ \frac{88}{3}S_2(N) - 32\tilde{S}(N) + 4S_3'\Big(\frac{N}{2}\Big) - \frac{17}{3} - \frac{88}{9}\pi^2 \\ &- \frac{4}{9}\,\frac{151N^4+368N^3+286N^2+69N+18}{N^3(N+1)^3} + 8\eta\,\frac{2N^2+2N+1}{N^3(N+1)^3}\Big] \\ + C_F T_R\Big[&-\frac{160}{9}S_1(N) - \frac{32}{3}S_2(N) + \frac{4}{3}S_2'\Big(\frac{N}{2}\Big) + \frac{32}{3}\pi^2 + 16 - \frac{11N^2+17N+3}{3N^2(N+1)^2}\Big]\end{aligned} \tag{B.58}$$

$$\bar{\gamma}_{qq}^{(1)N+1} = \bar{\gamma}_{NS}^{(1)N+1}(1) - 16 C_F T_R \frac{5N^5+7N^4-29N^3-33N^2-4N+4}{(N-1)^2 N^3 (N+1)^3 (N+2)} \tag{B.59}$$

$$\begin{aligned}\bar{\gamma}_{qg}^{(1)N+1} = & -8 C_A T_R \left[\left(2S_1^2(N) - 6S_2(N) + 2S_2'\left(\frac{N}{2}\right) + \frac{8\pi^2}{3} \right) \frac{N^2+N+2}{(N-1)N(N+1)} - 8S_1(N) \frac{2N-1}{(N-1)^2 N^2} \right. \\ & \left. - 2 \frac{N^9+3N^8+3N^7-4N^6-9N^5-49N^4-79N^3-62N^2-4N+8}{(N-1)^3 N^3 (N+1)^3 (N+2)} \right] \\ & + 8 C_F T_R \left[\left(2S_1^2(N) + 2S_2(N) + 5 + \frac{4\pi^2}{3} \right) \frac{N^2+N+2}{(N-1)N(N+1)} \right. \\ & \left. + 4 \frac{S_1(N)}{(N+1)^2} + \frac{11N^4+18N^3+3N^2-12N-4}{(N-1)N^3(N+1)^3} \right]\end{aligned} \tag{B.60}$$

$$\begin{aligned}\bar{\gamma}_{gq}^{(1)N+1} = & -4 C_F^2 \left[\left(2S_1^2(N) - 10S_1(N) - 2S_2(N) + \frac{8\pi^2}{3} \right) \frac{N^2+N+2}{N(N+1)(N+2)} - \frac{4S_1(N)}{N^2} \right. \\ & \left. + \frac{12N^6+42N^5+73N^4+84N^3+59N^2+20N+4}{N^3(N+1)^3(N+2)} \right] \\ & - 8 C_F C_A \left[\left(-S_1^2(N) - 3S_2(N) + S_2'\left(\frac{N}{2}\right) - \frac{8\pi^2}{3} \right) \frac{N^2+N+2}{N(N+1)(N+2)} \right. \\ & + S_1(N) \frac{17N^4+68N^3+143N^2+172N+68}{3N(N+1)^2(N+2)^2} \\ & \left. - \frac{109N^9+360N^8+356N^7-110N^6-1015N^5-1400N^4-410N^3+142N^2+168N+72}{9(N-1)^2 N^3 (N+1)^3 (N+2)^2} \right] \\ & - \frac{32}{3} C_F T_R \left[-\left(S_1(N) - \frac{8}{3} \right) \frac{N^2+N+2}{N(N+1)(N+2)} + \frac{1}{N^2} \right]\end{aligned} \tag{B.61}$$

$$\begin{aligned}\bar{\gamma}_{gg}^{(1)N+1} = & C_A T_R \left[-\frac{160}{9} S_1(N) + \frac{32}{3} + \frac{16}{9} \frac{38N^4+76N^3+94N^2+56N+12}{(N-1)N^2(N+1)^2(N+2)} \right] \\ & + C_F T_R \left[8 + 16 \frac{2N^6+8N^5+11N^4+14N^3+21N^2+20N+4}{(N-1)N^3(N+1)^3(N+2)} \right]\end{aligned} \tag{B.62}$$

+

$$+ C_A^2 \left[\frac{536}{9} S_1(N) - 64 S_1(N) \frac{2N^5+5N^4+8N^3+7N^2-2N-2}{(N-1)^2N^2(N+1)^2(N+2)^2} \right.$$

$$+ 32 \tilde{S}(N) - 4 S_3'\left(\frac{N}{2}\right) + 16 S_1(N) \left(4 S_2(N) - S_2'\left(\frac{N}{2}\right) - \frac{2\pi^2}{3}\right)$$

$$- \frac{64}{3} - 32 \left(4 S_2(N) - S_2'\left(\frac{N}{2}\right) - \frac{2\pi^2}{3}\right) \frac{N^2+N+1}{(N-1)N(N+1)(N+2)}$$

$$\left. - \frac{4}{9} \frac{457N^9+1371N^8+556N^7-2206N^6-4151N^5-4249N^4-2310N^3-676N^2+552N+288}{(N-1)^3N^3(N+1)^3(N+2)^2} \right]$$

The violation of the Gribov-Lipatov reciprocity relation can be summarized by the differences

$$\Delta K^N = K^N - \bar{K}^{N+1} \tag{B.63}$$

or

$$\Delta K(x) = K(x) - \frac{1}{x} \bar{K}\left(\frac{1}{x}\right) \tag{B.64}$$

In what concerns the second order non singlet quantities this violation reads as

$$\Delta C_{2,q}^{(1)N} = C_F \left[-4 S_2(N) + (2N+1) \frac{7N^2+7N+2}{N^2(N+1)^2} - \frac{4}{3}\pi^2 \right] \tag{B.65}$$

$$\Delta \gamma_{NS}^{(1)N} = 16 C_F^2 \left[\left(4 S_1(N) - \frac{3N^2+3N+2}{N(N+1)}\right) \frac{2N+1}{2N^2(N+1)^2} - \left(2 S_1(N) - \frac{1}{N(N+1)} - \frac{3}{2}\right)\left(2 S_2(N) - \frac{\pi^2}{3}\right) \right]$$

$$- \frac{8}{3}(11 C_A - 4 T_R) C_F \left[2 S_2(N) - \frac{\pi^2}{3} - \frac{2N+1}{N^2(N+1)^2} \right] \tag{B.66}$$

$$\Delta e_{2,q}^{(1)}(x) = C_F \left[7(1+x) - 2 \frac{1+x^2}{1-x} \log x - 2\pi^2 \delta(1-x) \right] \tag{B.67}$$

$$\Delta P_{NS}^{(1)}(x) = -8 C_F^2 \left[\left(-5-x+\frac{6}{1-x}\right) \log x + \left(3+3x-\frac{4}{1-x}\right) \log^2 x + 4 \frac{1+x^2}{1-x} \log x \log|1-x| \right]$$

$$+ \frac{8}{3}(11 C_A - 4 T_R) C_F \frac{1+x^2}{1-x} \log x \tag{B.68}$$

FOOTNOTES

f 1 In the T.L. region one can work also with a variable $z = 1/x$, $0 < z < 1$. In this case eq. (4.3) takes the form

$$\bar{K}(z) \otimes \phi(z) = K^{[1]}\left(\frac{1}{z}\right) \phi(z) - \int_z^1 \frac{dy}{y} \phi\left(\frac{z}{y}\right) \bar{K}^{[2]}\left(\frac{1}{y}\right) - \int_z^1 \frac{dy}{y} \left(\phi\left(\frac{z}{y}\right) - \frac{\phi(z)}{y}\right) \bar{K}^{[3]}\left(\frac{1}{y}\right)$$

where $\bar{K}^{[i]}$ are the kernels given in sect. 4 and in Appendix B.

f 2 Some authors write rel. (5.1) in the form $\tilde{F}\left(\frac{1}{x_T}\right) = x_S F(x_S)$ where $x_T = 1/x_S = 1/x$.

f 3 Some authors, [10], write for eq. (5.3) $\tilde{F}(x_T, Q^2) = F(x_S, Q^2)$ where $x_T = 1/x_S = 1/x$.

f 4 In ref. [16] the parameter θ takes the value $\theta = -1$ in the T.L. region and not $\theta = +1$.

f 5 We warn the reader of an inconsistency of conventions in ref. [23] . Once the conventions of Gross and Wilzeck (Phys. Rev. D9 (1974) 980) are taken for the off-diagonal elements of the first-order anomalous dimension matrix (ADM), then one has to change the signs of the off-diagonal elements of the second-order ADM and of the gluon coefficient function of ref. [10, 23] . See also Duke and Roberts (Nucl. Phys. B166 (1980) 243).

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FIGURE CAPTIONS

Fig. 1 : (a) Parton a decaying into parton a' in the process $\gamma^* \rightarrow$ hadron + anything.
(b) Parton a found in parton a' in the process $\gamma +$ hadron $\rightarrow$ anything.

Fig. 2 : Cut vertices as $Q^2 \rightarrow \infty$, x fixed, limits of tree Compton type diagrams and their Feynman rules.

Fig. 3 : Diagrams necessary for the calculation of the α corrections to the coefficient functions $e(x,\alpha)$ and $\bar{e}(x,\alpha)$.

Fig. 4 : Diagrams necessary for the calculation of the α^2 corrections to the anomalous dimension kernels $P_{NS}(x,\alpha)$ and $\bar{P}_{NS}(x,\alpha)$.

Fig. 5 : Diagram with simple and double virtual and real gluon emission.

Fig. 6 : Diagram with simple real and virtual gluon emission.

Fig. 7 : Possible cuts for the diagram of the calculation example.

Fig. 8 : Comparison of time-like (T) and space-like (S) integrated evolution second-order factor in the moment space. The dotted curve represents the extraction of the π^2 term from the time-like factor.

Fig. 9 : The same as in fig. 8 but separate comparison for (a) the coefficient functions (b) anomalous dimensions and (c) local in Q^2 evolution kernel in the moment space.

Fig. 10 : Comparison of the contributions to the local derivative of an input non singlet structure function eq. (6.6) and inclusive annihilation fragmentation function eq. (6.7) (Positive solid curve). They are chosen to be Gribov-Lipatov symmetric. The continuous curve is the first order contribution in the S.L.

and T.L. regions, the dashed (dotted) curves include the α^2-corrections in the S.L. (T.L.) region.

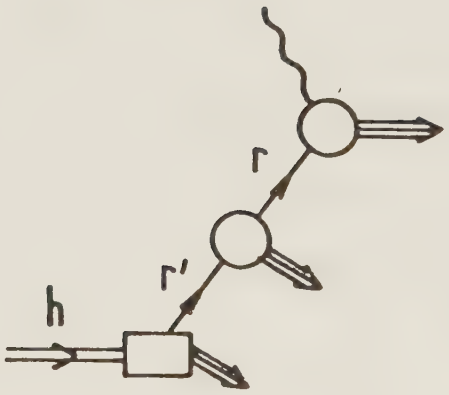
Fig. 11 : The evolution from Q = 3 GeV for input non-singlet functions (a) eq. (6.6) (S.L. region) and (b) eq. (6.7) (T.L. region) to Q = 10, 30 GeV. We have taken Λ = .5 GeV/c.

Fig. 12 : First-order local derivatives for space-like (solid line) and time-like (dotted line) regions and first plus second-order derivatives for space-like (dashes) and time-like (open circles) regions. Positive solid line represents the input structure or fragmentation function , in the same units , given by eq. (6.10). Fig. (a) for $x F_{2,s}$, singlet and (b) for $x G(x)$ gluon distributions.

Fig. 13 : Evolution of the $x F_{2,s}$ (fig. (a)) and $\frac{1}{x^2} \overline{F}_{2,s}$ (fig. (b)) singlet structure and fragmentation functions from Q = 3 GeV to Q = 10, 30 GeV, with input functions from eq. (6.10).

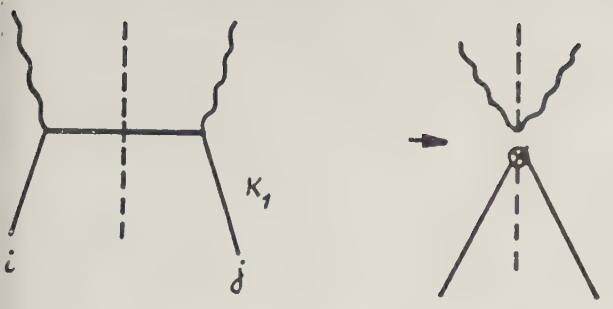
Fig. 14 : Same evolution as in fig. 13 for $x G$ and $\frac{1}{x^2} \overline{G}$.

(a)

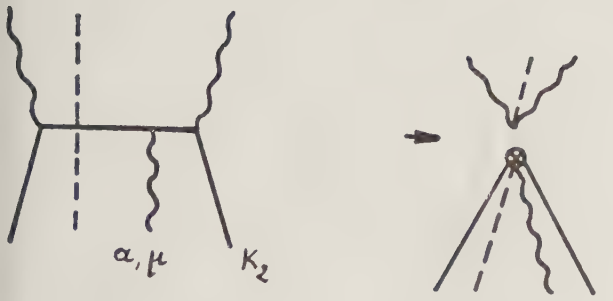


(b)

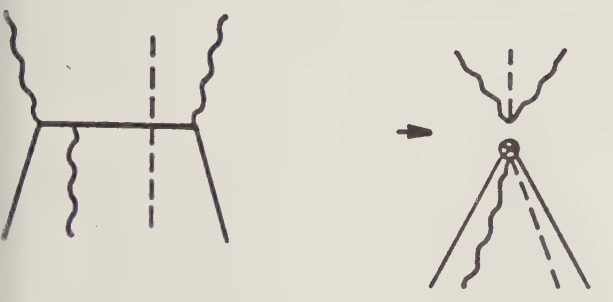
Fig. 1



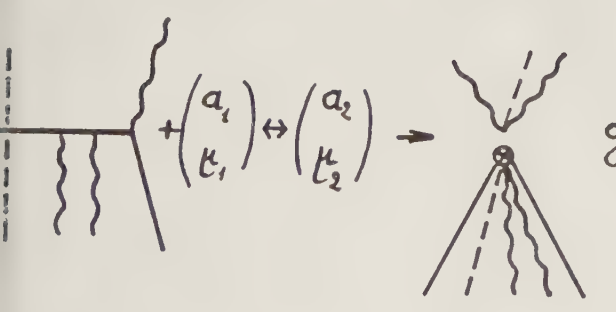
$$\delta_{ij} \not\propto \delta(\rho_1 - x)\,\theta(\rho_1)$$



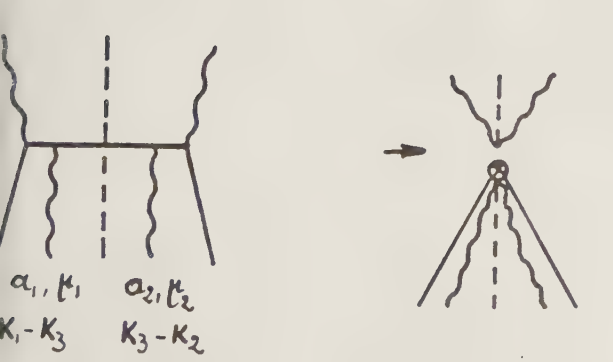
$$g\,t^a_{ij} \not\propto \Delta_\mu \frac{\delta(\rho_1 - x)}{\rho_1 - \rho_2}\,\theta(\rho_1)$$



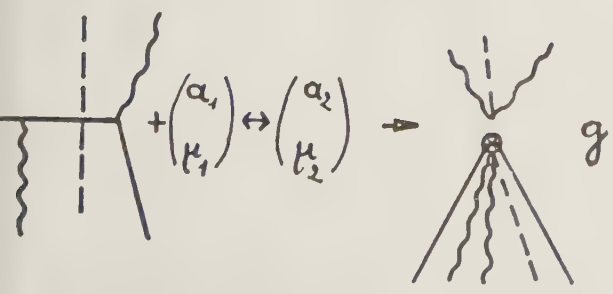
$$-g\,t^a_{ij} \not\propto \Delta_\mu \frac{\delta(\rho_2 - x)}{\rho_1 - \rho_2}\,\theta(\rho_2)$$



$$g^2 \not\propto \Delta_{\mu_1}\,\Delta_{\mu_2} \frac{\delta(\rho_1 - x)\,\theta(\rho_1)}{1 - \rho_1}\left[\frac{(t^{a_1}t^{a_2})_{ij}}{1-\rho_3} + \frac{(t^{a_2}t^{a_1})_{ij}}{\rho_3 - \rho_1}\right]$$



$$-g^2 \not\propto \Delta_{\mu_1}\,\Delta_{\mu_2} \frac{\delta(\rho_3 - x)\,\theta(\rho_3)}{(1-\rho_3)(\rho_3 - \rho_1)}\,(t^{a_2}t^{a_1})_{ij}$$



$$g^2 \not\propto \Delta_{\mu_1}\,\Delta_{\mu_2} \frac{\delta(\rho_2 - x)\,\theta(\rho_2)}{1 - \rho_2}\left[\frac{(t^{a_1}t^{a_2})_{ij}}{\rho_3 - \rho_1} + \frac{(t^{a_2}t^{a_1})_{ij}}{1-\rho_3}\right]$$

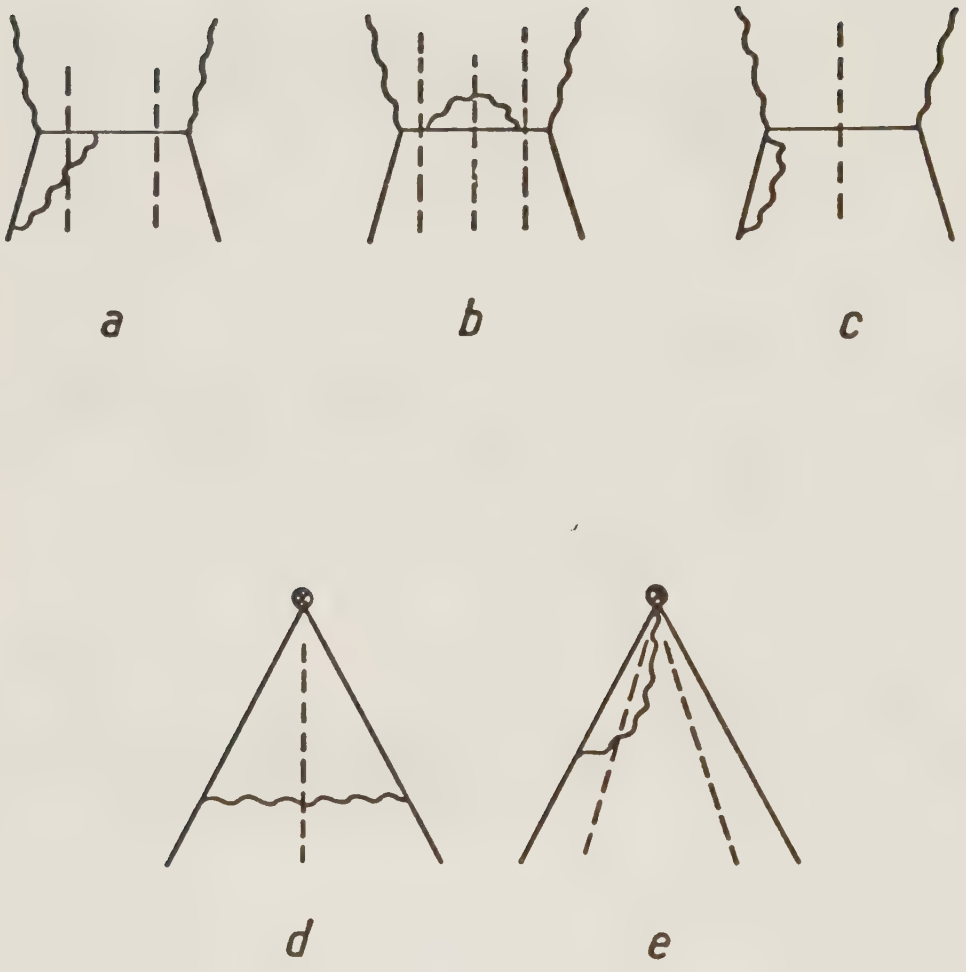


Fig _ 3

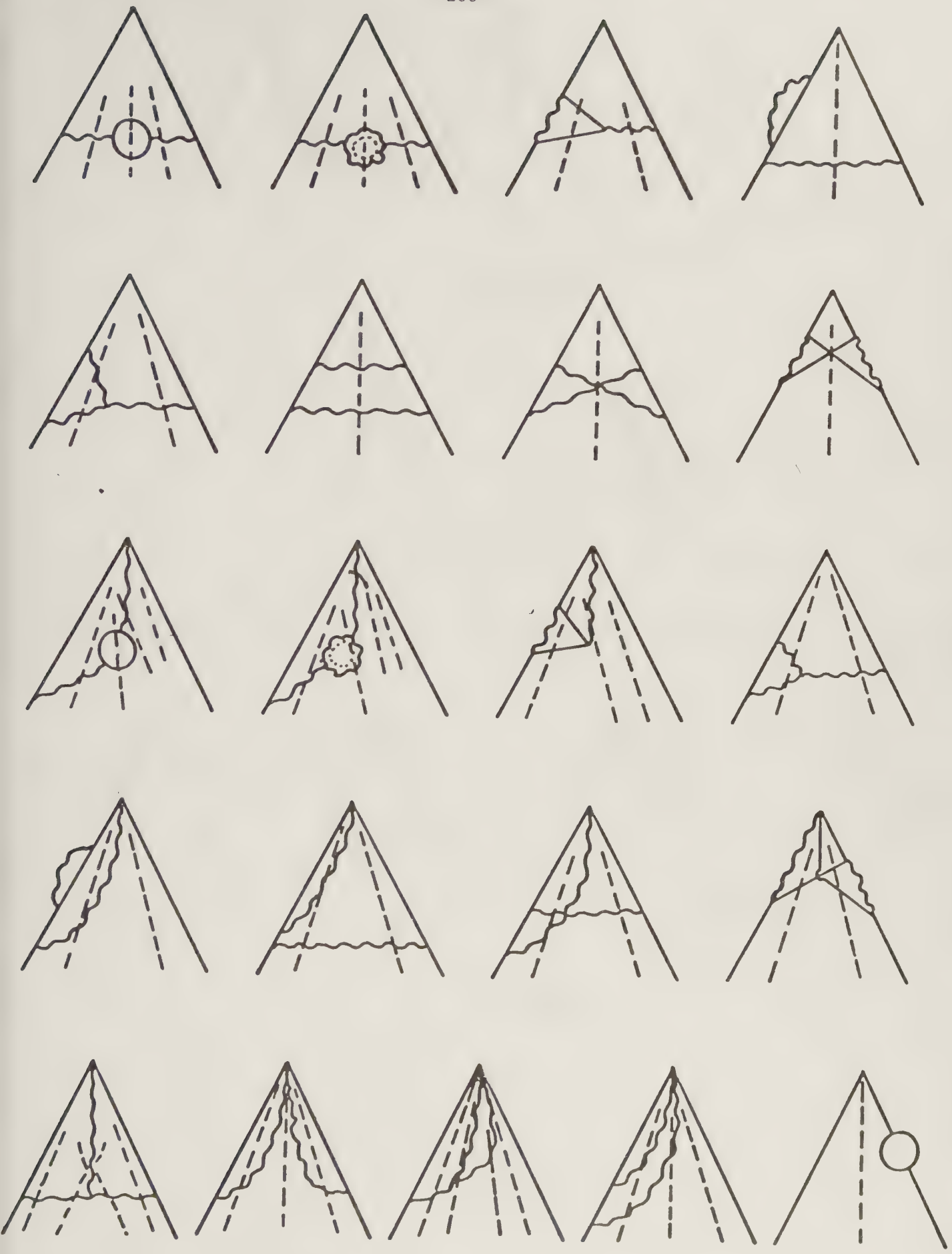


Fig - 4

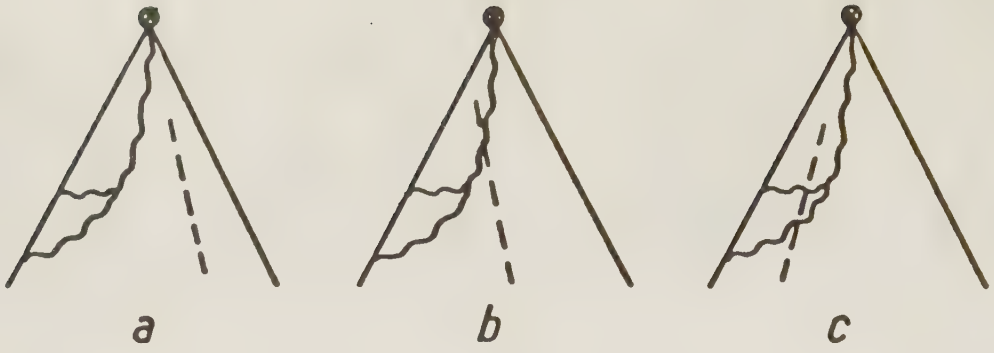


Fig. 5

Fig. 6

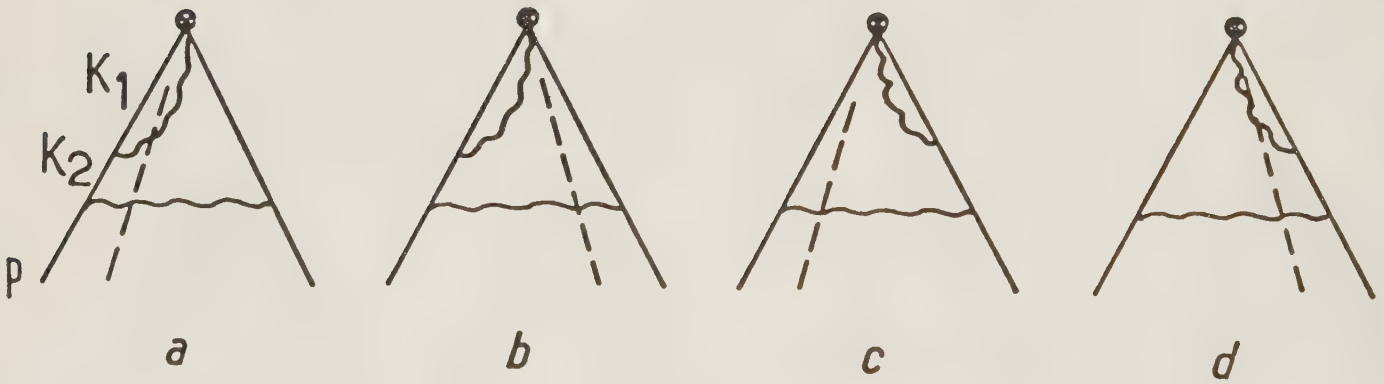


Fig. 7

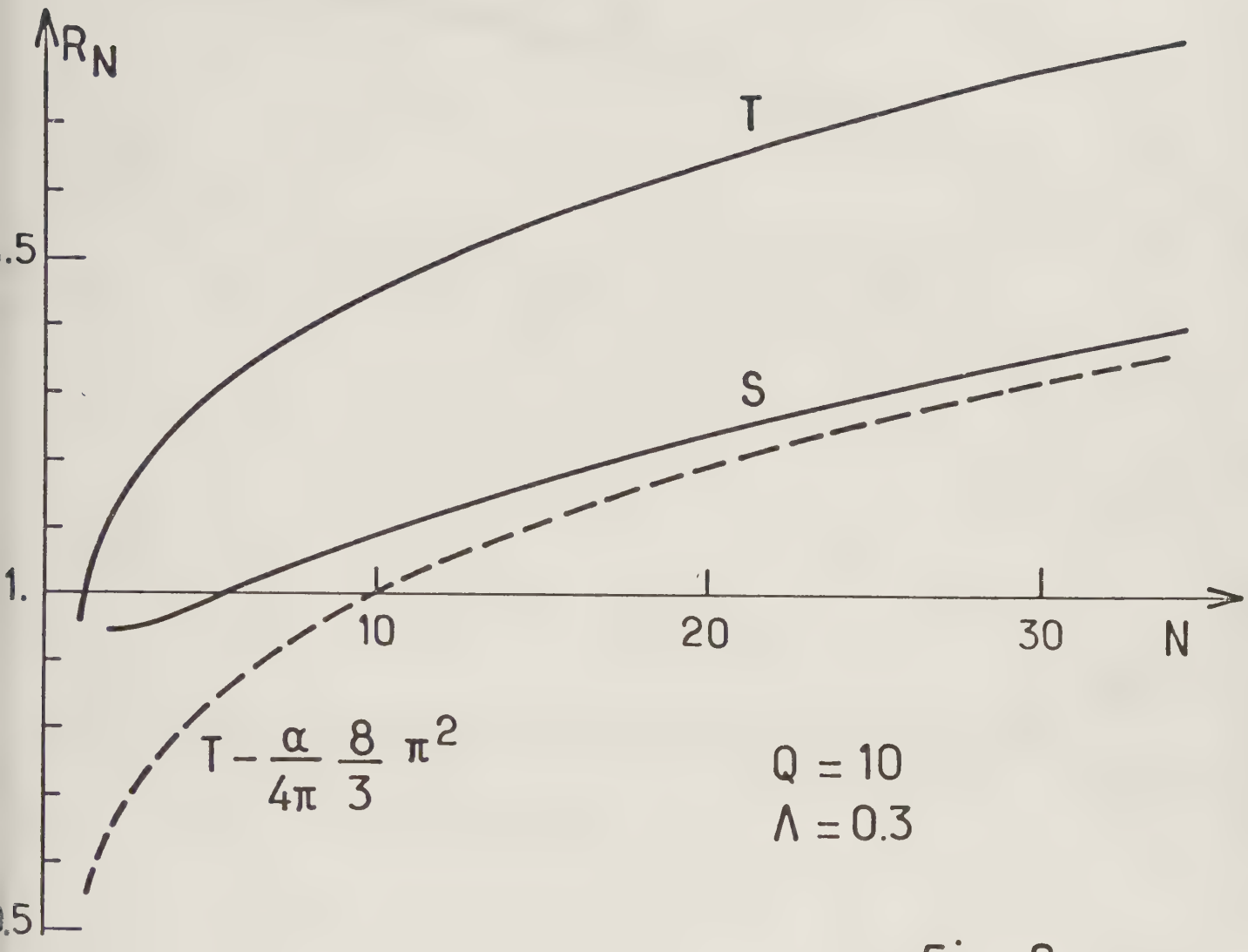


Fig - 8

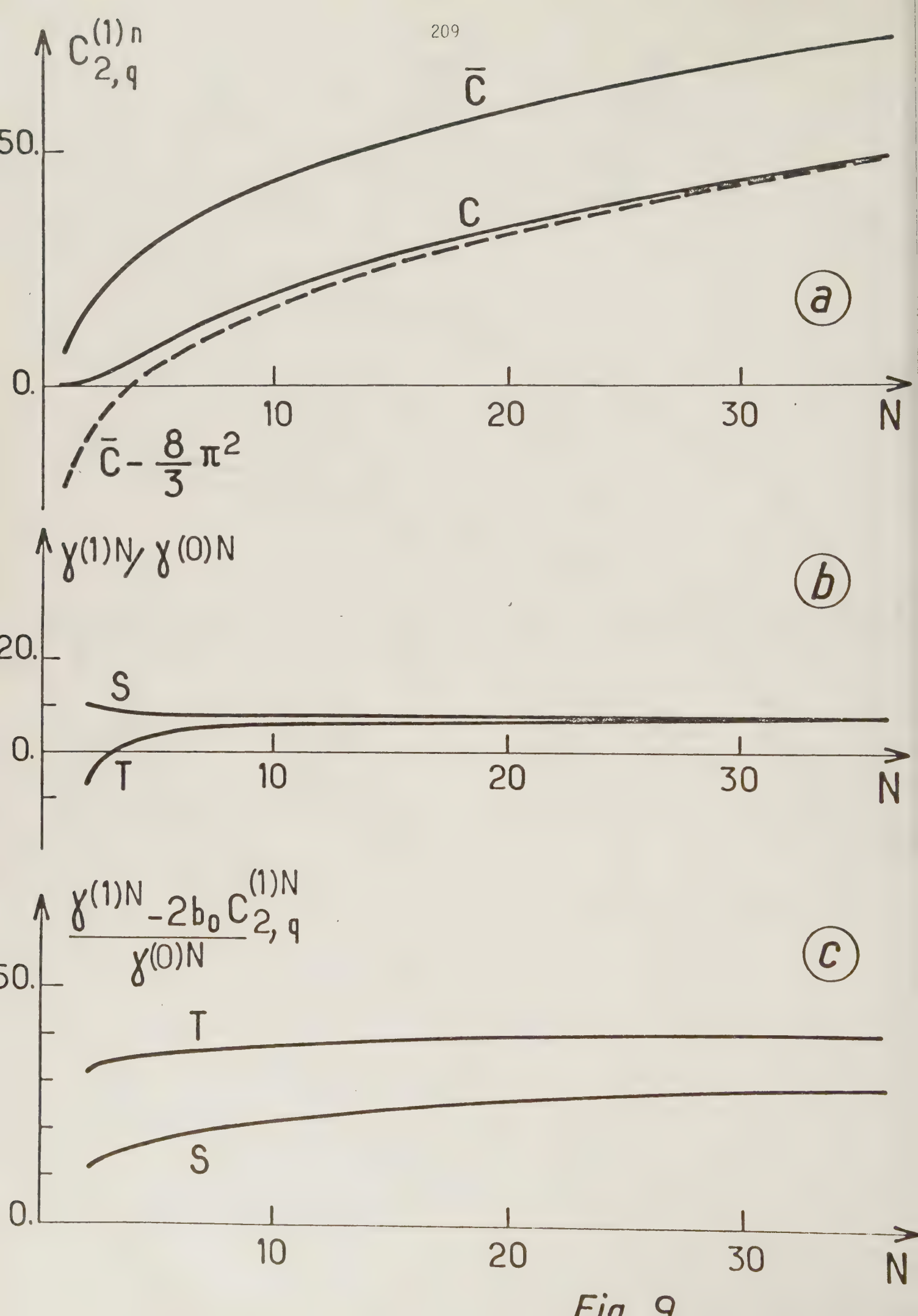

$C^{(1)n}_{2,q}$
50.
0.
$\bar{C}$
C
a
10
20
30
N
$\bar{C} - \frac{8}{3}\pi^2$
$\gamma^{(1)N}/\gamma^{(0)N}$
b
20.
0.
S
T
10
20
30
N
$\frac{\gamma^{(1)N} - 2b_0 C^{(1)N}_{2,q}}{\gamma^{(0)N}}$
c
50.
T
S
0.
10
20
30
N

Fig. 9

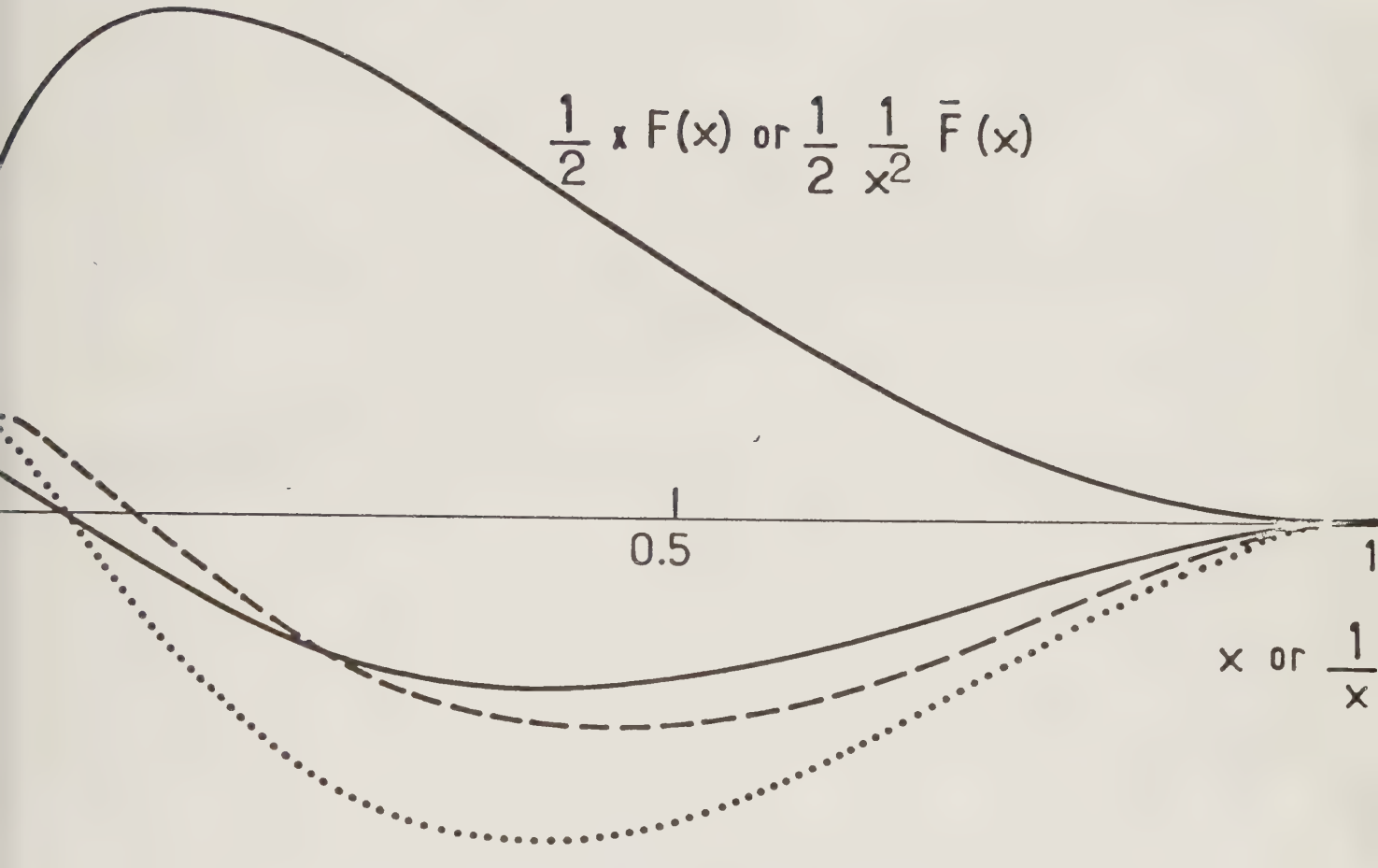


Fig_10

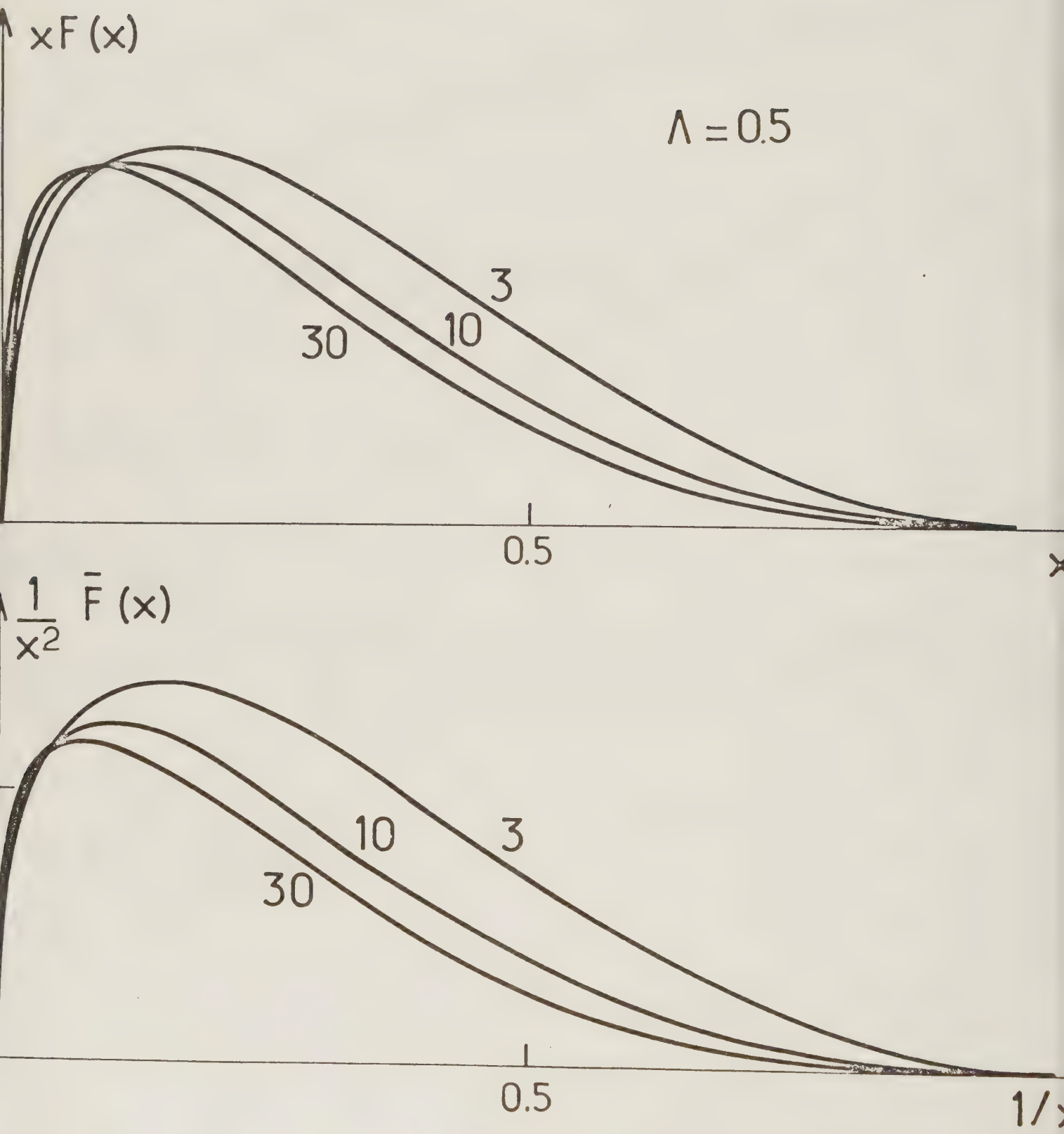


Fig. 11

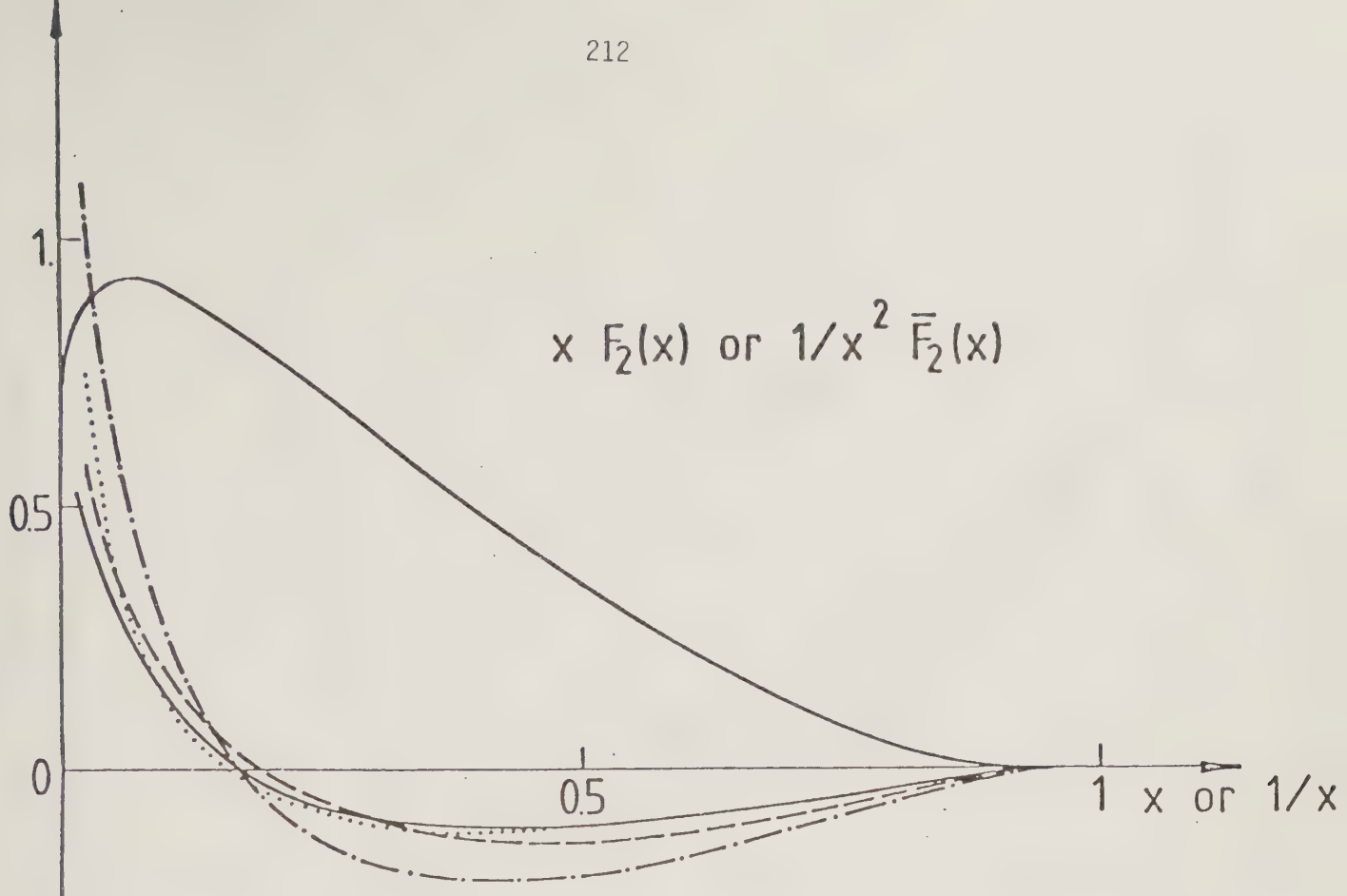


Fig 12 a

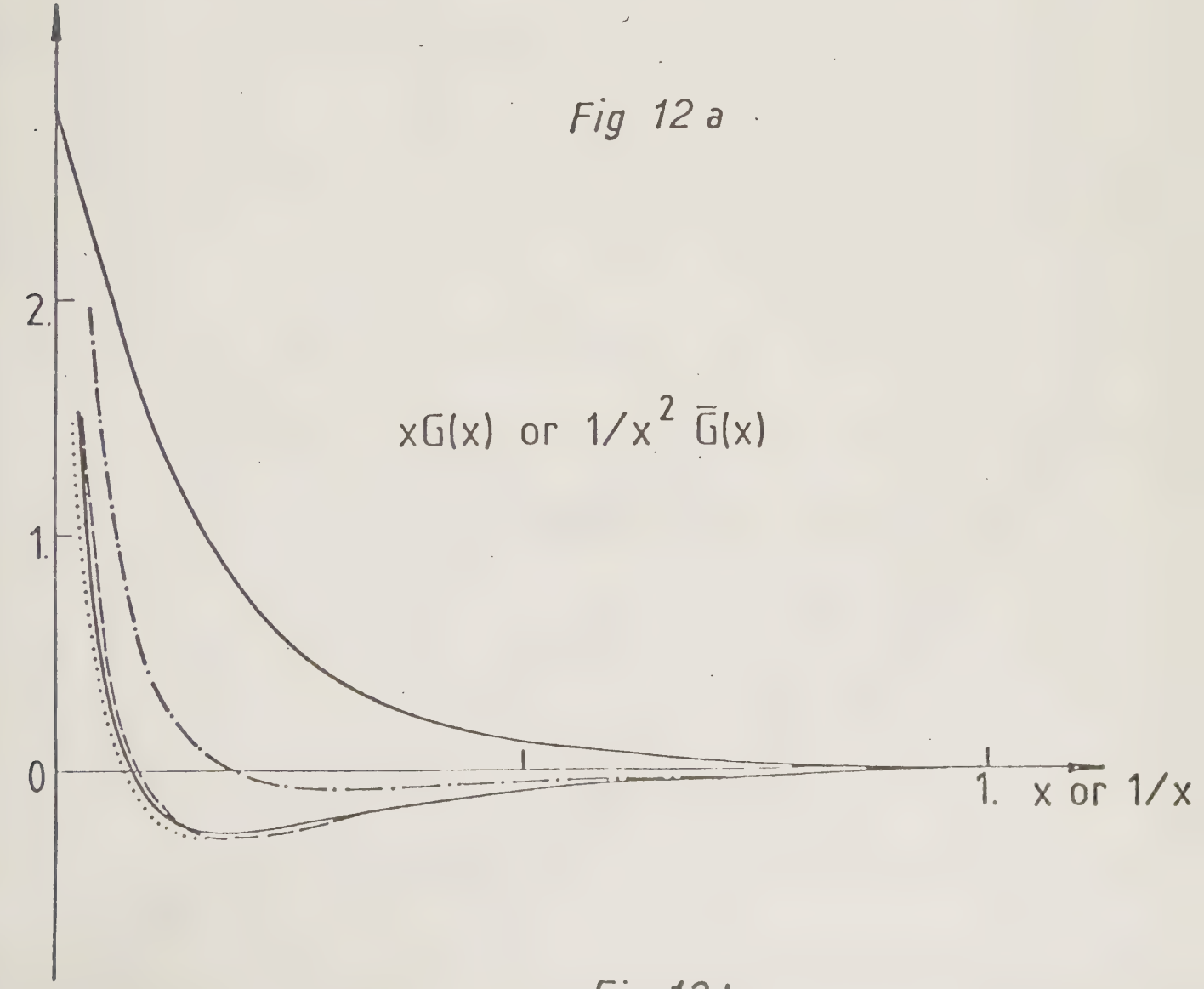


Fig 12 b

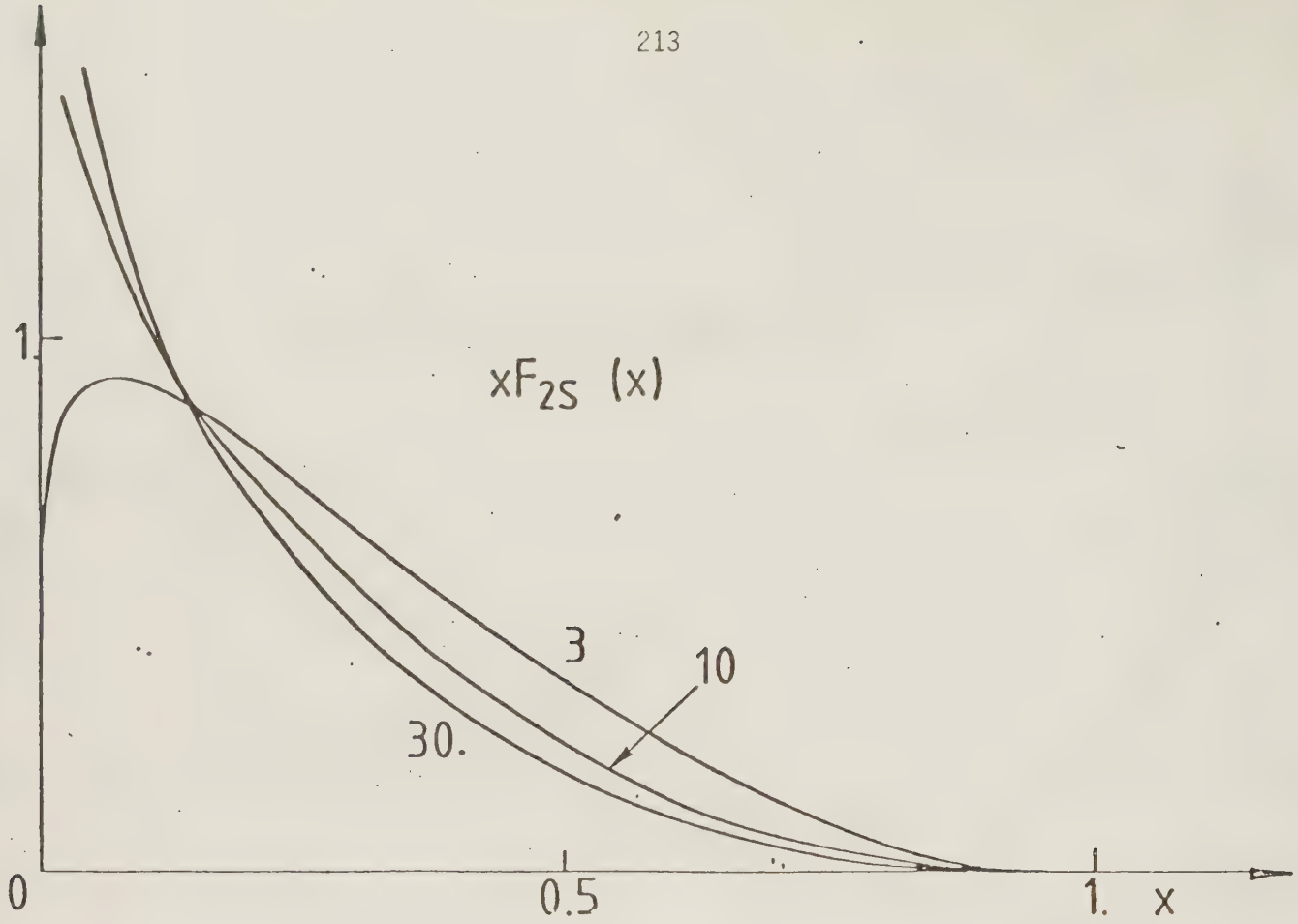
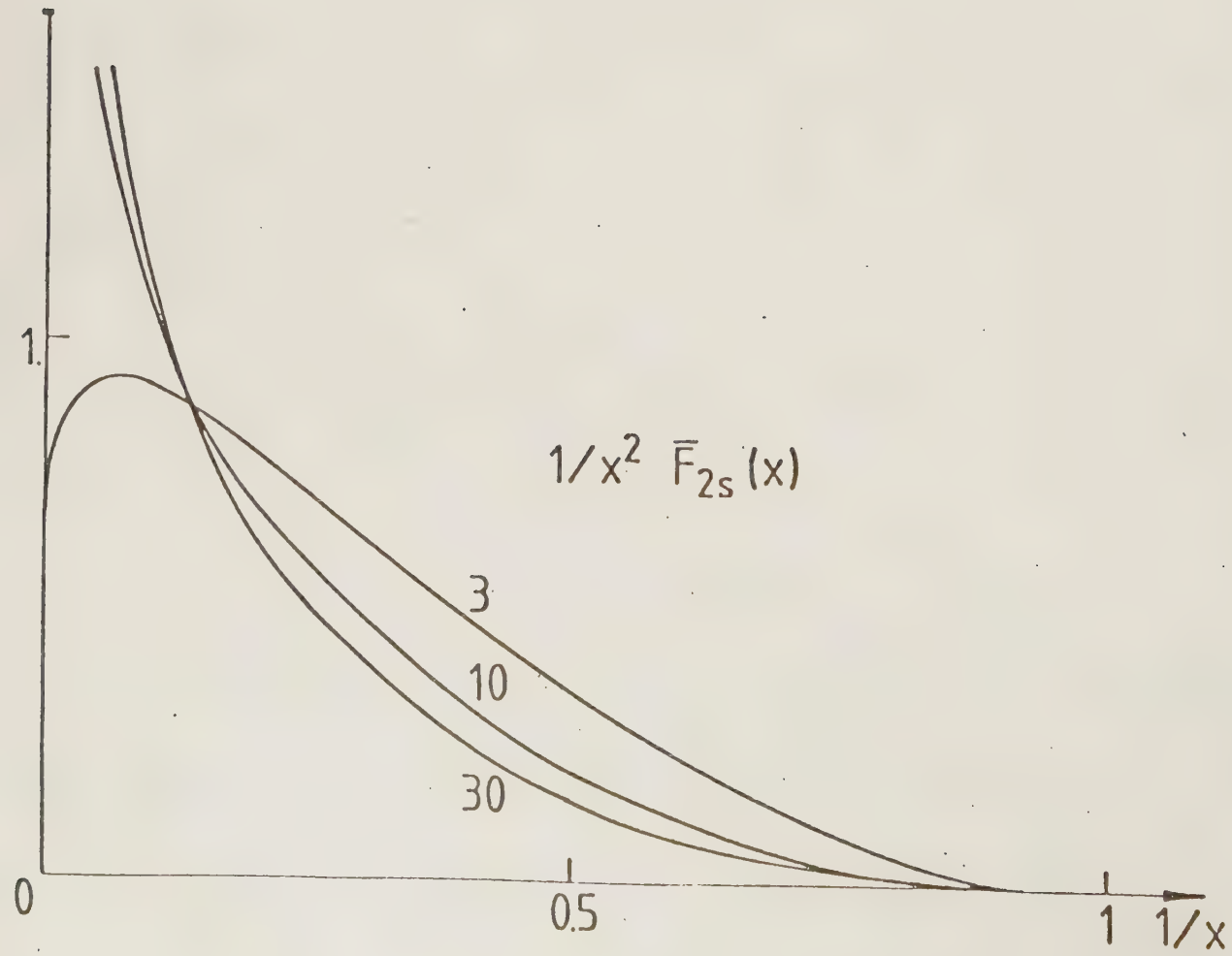


Fig 13 a



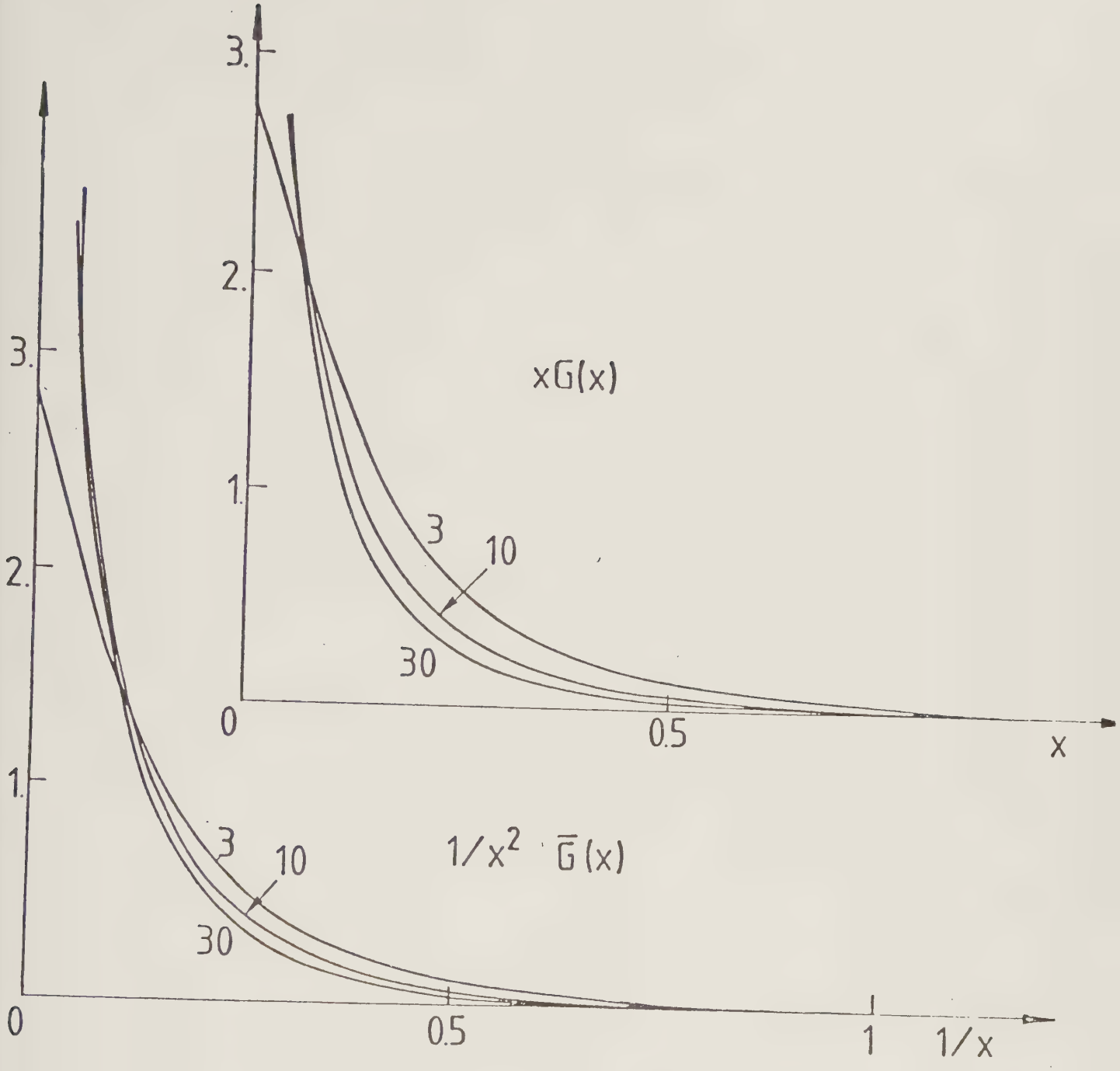


Fig. 14

SOLUTION ANALYTIQUE

DANS L'ESPACE DES X

Dans l'espace des moments les équations différentielles du modèle des partons généralisé ont une solution analytique simple. Dans le cas non-singulet elles s'écrivent [1]

$$\langle F(x,Q)\rangle_\sigma = \langle F(x,Q_0)\rangle_\sigma \exp\left[\frac{\langle K^0(x)\rangle_\sigma}{-2b_0}\left(S(Q,Q_0) + \frac{\xi(Q,Q_0)}{4\pi}\left(\frac{\langle K^1(x)\rangle_\sigma}{\langle K^0(x)\rangle_\sigma} - \frac{b_1}{b_0}\right)\right)\right] \qquad (1)$$

où $\langle F(x,Q)\rangle_\sigma$ représente le moment σ de fonction $F(x,Q)$, (+)

$$S(Q,Q_0) = \log\frac{\alpha(Q_0)}{\alpha(Q)} \qquad (2a)$$

$$\xi(Q,Q_0) = \frac{4\pi b_0}{b_1}\log\left(\frac{1+\frac{b_1}{4\pi b_0}\alpha(Q_0)}{1+\frac{b_1}{4\pi b_0}\alpha(Q)}\right) \simeq \alpha(Q_0)-\alpha(Q) \qquad (2b)$$

La constante de couplage effective $\alpha(Q)$ s'exprime en fonction du paramètre Λ :

$$\frac{4\pi}{\alpha(Q)} = -b_0\log\frac{Q^2}{\Lambda^2} + \frac{b_1}{b_0}\log\log\frac{Q^2}{\Lambda^2} \qquad (3)$$

Dans le cas de $SU(3)_c$ avec 4 saveurs, on a $b_0 = -\frac{25}{3}$ et $b_1/b_0 = \frac{154}{25}$. $K^0(x)$ ainsi que $K^1(x)$ sont donnés au chapitre précédent. [2]

L'équation (1) est très simple, mais elle présente l'inconvénient d'être exprimée dans l'espace des moments. Du point de vue expérimental, cela pose un grand problème, car les moments de la fonction de structure sont très sensibles aux extrapolations faites pour les petites et grandes valeurs de X, c'est à dire justement là où la statistique sur $F(x,Q)$ est insuffisante. [3]

Il serait donc utile d'exprimer d'une manière aussi simple les prédictions théoriques dans l'espace des variables X. Cela s'avère possible pour

---

(+) $\langle\varphi(x)\rangle_\sigma = \int_0^1 dx\, x^{\sigma-1}\varphi(x)$ pour la diffusion;

$\langle\tilde\varphi(x)\rangle_\sigma = \int_0^\infty dx\, x^{-\sigma-1}\tilde\varphi(x)$ pour l'annihilation.

certaines paramétrisations des fonctions de structures. Les formes les plus utilisées pour les fonctions de structure étant les suivantes:

$$\text{(a)} \quad F(x,Q) = C\, x^{\alpha}(1-x)^{\beta}\left(1+\frac{\gamma}{\beta+1}(1-x)\right) \qquad x_{min} < x < x_{max} \tag{4a}$$

$$\text{(b)} \quad F(x,Q) = C'\, x^{\alpha}\log^{\beta}\frac{1}{x}\left(1+\frac{\gamma}{\beta+1}\log\frac{1}{x}\right) \qquad x_{min} < x < x_{max} \tag{4b}$$

Nous déterminerons les prédictions théoriques pour la dépendance en Q des paramètres α, β, γ et C'.

Les paramétrisations de $F(x,Q)$ eq.(4a et 4b) permettent d'effectuer analytiquement la transformation de Mellin inverse de $\log \langle F(x,Q)\rangle_{\sigma}$. Grâce à cette propriété d'inversion, et à l'aide de l'équation (1), nous pouvons obtenir une équation qui exprime les prédictions théoriques dans l'espace des X sans faire intervenir les complications du produit de convolution.

$$\varphi(x,Q) = \varphi(x,Q_0) + S(Q,Q_0)\frac{K^{o}(x)}{-2b_0} + \frac{S(Q,Q_0)}{4\pi}\left[\frac{K'(x)-\frac{b_1}{b_0}K^{o}(x)}{-2b_0}\right] \tag{5}$$

où $\varphi(x,Q)$ est la transformée de Mellin inverse de la fonction $\log \langle F(x,Q)\rangle_{\sigma}$. Elle est égale à

$$\varphi_a(x,Q) = \frac{1}{(1-x)\left(\log\frac{1}{x}\right)_+}\left[x^{\alpha} - x^{\alpha+\beta+2} - (1-x)\,x^{\alpha+\beta+\gamma+1}\right] + \log\left(\Gamma(\beta+1).C\right).\delta(1-x) \tag{6a}$$

dans le cas de l'expression (4a), et elle devient

$$\varphi_b(x,Q) = \frac{1}{\left(\log\frac{1}{x}\right)_+}\left[(\beta+2)\,x^{\alpha} - x^{\alpha+\gamma}\right] + \log\left(\Gamma(\beta+1).C\right).\delta(1-x) \tag{6b}$$

dans le cas de l'expression (4b).

L'équation (5) peut être vérifiée expérimentalement pour x compris entre x_{min} et x_{max} . On ne peut pas vérifier si le paramètre $C(Q)$ est en accord avec les prédictions théoriques, puisque la dépendance de $f(x,Q)$ en $C(Q)$ est proportionnnelle à une fonction $\delta(1-x)$, qui donne une contribution nulle dans la région (x_{min} , x_{max}).

On peut éviter ce problème en utilisant des règles de somme pour les fonctions de structure non-singulet. En effet, le premier moment des fonctions de structure est toujours connu avec une normalisation bien définie, relié aux nombres quantiques de la cible hadronique. On a par exemple

$$\frac{1}{3}\int_0^1 dx\ F_3^{\nu N+\bar{\nu}N}(x,Q) = \left[1 + 0.32\,\alpha(Q) + \mathcal{O}(\alpha^2(Q)\right] \qquad (7)$$

A partir de l'équation (5), on peut déterminer le paramètre Λ en utilisant les données expérimentales pour α , β , γ aux différents Q. Il faut faire très attention aux corrélations statistiques possibles entre C et α , β , γ . Des corrélations de ce type sont présentes pour les fonctions de structure expérimentales. Les règles de somme (7) permettent d'éviter ce problème des corrélations en laissant trois paramètres libres α , β , γ , permettant ainsi d'obtenir une très bonne paramétrisation des résultats expérimentaux pour les fonctions de structure.

Une autre façon de procéder est d'exprimer α , β et γ en fonction de Q , étant données des valeurs initiales α_0 , β_0 , γ_0 à Q_0 , en utilisant l'équation (5). La valeur $C(Q)$ s'exprime en fonction de α, β et γ à l'aide de la règle de somme (7) :

$$C = \frac{\alpha+\beta+2}{\alpha+\beta+\gamma+1} \cdot \frac{1}{B(\alpha+1,\beta+1)} \cdot \left(1 - 0.32\,\alpha(Q)\right) \qquad (8a)$$

dans le cas de la paramétrisation (4a), et :

$$C = \frac{(\alpha+1)^{\beta+2}}{(\alpha+\gamma+1)\,\Gamma(\beta+1)} \cdot \left(1 - 0.32\,\alpha(Q)\right) \qquad (8b)$$

dans le cas de la paramétrisation (4b).

Nous donnons ici les quantités qui permettent de faire cette analyse, dans le cas de $SU(3)_c$ et $SU(4)$ saveur

$$f^0(x) \equiv \frac{k^0(x)}{-2b_0} = \frac{8}{25}\left(\frac{1+x^2}{1-x}\right) \quad ; \quad x<1 \tag{9a}$$

$$f_c^{(1)}(x) \equiv \left(\frac{k^1(x)}{k^0(x)} - \frac{b_1}{b_0}\right) = \frac{1}{3}\cdot\frac{1}{1+x^2}\Big\{32.078 - 103x + 119.578x^2$$
$$+ 4(11-4x+12x^2)\log x + (5+13x^2)\log^2 x - 25(1+x^2)\log|1-x|$$
$$- 16(1+x^2)\log x\log|1-x| - c\Big[4(1-x)^2 - 2(1-x^2)\log x$$
$$+ \frac{1-x}{1+x}(1+x^2)\Big(\log^2 x - 4\log x\log(1+x) - 4\,Li_2(-x) - \frac{\pi^2}{3}\Big)\Big]\Big\} \quad ; \quad x<1. \tag{9b}$$ (+)

Dans le cas d'analyse de la fonction de structure $\frac{1}{x}F_2^{NS}(x,Q)$:

$$f_2^+(x) = f_{c=+1}^{(1)}(x). \tag{10a}$$

Dans le cas d'analyse de la fonction de structure $F_3^{NS}(x,Q)$, avec $c=\pm$:

$$f_3^{\pm}(x) = f_{\pm}^{(1)}(x) + \frac{25}{3}\frac{1-x^2}{1+x^2} \tag{10b}$$

Les règles de sommes, pour les fonctions de structures non-singulet, sont les suivantes:

$$\int_0^1 dx\; F_3^{\pm}(x,Q) = c_3\left[1 - \frac{\alpha(Q)}{\pi}(1 - 0.019(1\pm 1))\right] \tag{11a}$$

$$\int_0^1 dx\; F_2^{\pm}(x,Q) = c_2\left[1 + \frac{\alpha(Q)}{\pi}0.019\,(1\pm 1)\right] \tag{11b}$$

(+) Une bonne approximation de $Li_2(-x)$, $0\le x\le 1$ est la suivante

$$Li_2(-x) = -\tfrac{1}{2}\log^2(1+x) - \frac{1}{1+2x}\left[-2\log(1+x) + 3x + \frac{1}{4}\frac{x^2}{1+x} + \frac{1}{36}\frac{x^3}{(1+x)^2} + \frac{1}{144}\frac{x^4}{(1+x)^3} + \dots\right]$$

dans le cas de diffusion où $x>1$ on utilise la propriété

$$Li_2(-x) = -Li_2\left(-\frac{1}{x}\right) - \frac{\pi^2}{6} - \frac{1}{2}\log^2 x$$

La table I donne les valeurs de $f_2^{\pm}$, $f_3^{\pm}$ pour differentes valeurs de X, et la figure (1) représente $f_2^{+}(x)$ comme une fonction de X.

Dans le cas d'annihilation, comme conséquence de la continuation analytique, on trouve que :

$$\tilde{f}^{0}(x) = - f^{0}(x) \qquad x > 1 \tag{12a}$$

et

$$\tilde{f}_c^{(1)}(x) = f_c^{(1)}(x) \qquad x > 1 \tag{12b}.$$

La violation de la relation de Gribov et Lipatov s'exprime par la différence

$$\Delta_i(x) = \frac{1}{-2b_0}\left[x\tilde{K}_i^{1}\left(\frac{1}{x}\right) - K_i^{1}(x)\right] \equiv f^{0}(x) \cdot \Delta f_i(x) - \frac{8}{3}\pi^2\delta(1-x) \tag{13a}$$

où

$$\Delta f_3(x) = \frac{1}{3}\,\frac{1}{1+x^2}\left[\frac{75}{2}(1-x^2) - (67-32x+67x^2)\log x - 8(1+3x^2)\log^2 x + 32(1+x^2)\log x \log(1-x)\right] \tag{13b}$$

$$\Delta f_2(x) = \Delta f_3(x) + \frac{50}{3}\,\frac{1-x^2}{1+x^2} \tag{13c}.$$

Nous voyons dans la formule (13a) qu'il y a deux types de termes violant la relation de Gribov-Lipatov. Le terme $-\frac{8}{3}\pi^2\delta(1-x)$ agit essentiellement sur la normalisation de la fonction de structure et de fragmentation (coefficient C and $\tilde{C}$ dans les formules (4) et (6)). L'autre terme $f^{0}(x)\,\Delta f_i(x)$ induit une différence dans l'évolution avec Q des coefficients $\tilde{\alpha}$, $\tilde{\beta}$ et $\tilde{\gamma}$ entre les cas de diffusion et d'annihilation. Les valeurs de Δf_i sont données dans la Table II et les $\tilde{f}_2^{\pm}(1/x)$, $\tilde{f}_3^{\pm}(1/x)$ sont données dans la Table III. On peut voir que les $\Delta f_i(x)$ sont très grandes et positives pour les petites valeurs des X. Les effets du second ordre sont beaucoup plus importants pour le processus d'annihilation que pour celui de diffusion comme cela apparaît sur la figure (1).

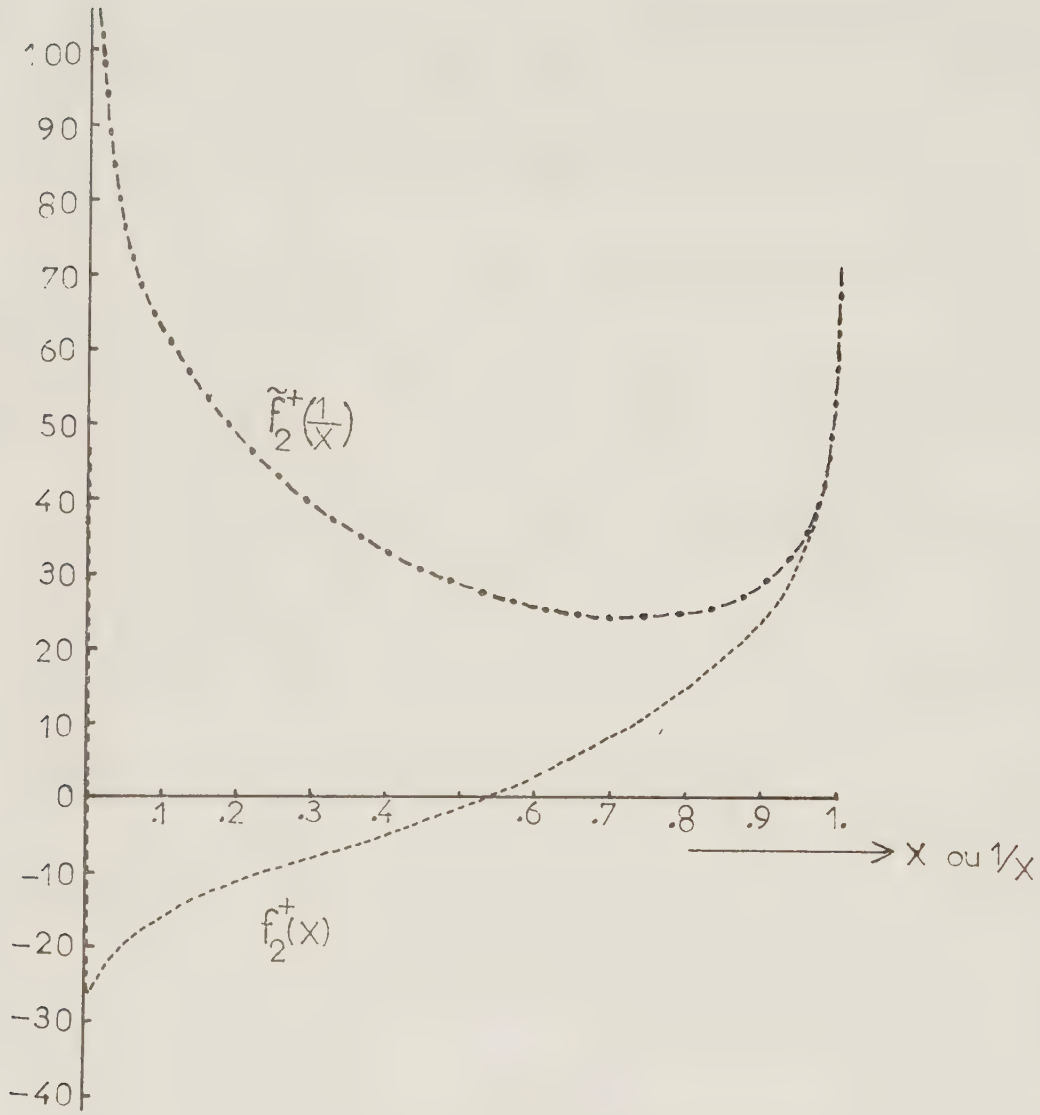


Fig. 1 : La correction du second ordre

(a) dans le cas de diffusion

$$f_2^+(x) = \frac{K_2'(x) - b_1 K^0(x)}{b_0 K^0(x)}$$

(b) dans le cas d'annihilation

$$\tilde{f}_2^+(1/x) = \frac{\tilde{K}_2'(1/x) - b_1 \tilde{K}^0(1/x)}{b_0 \tilde{K}^0(1/x)}$$

La relation de Gribov et Lipatov $f_2^+(x) = \tilde{f}_2^+(1/x)$ est violée fortement pour les petites valeurs des X.

TABLE I

X	f_2^+	f_2^-	f_3^+	f_3^-
10^{-5}	26.01	99.50	34.34	107.83
10^{-4}	- 7.33	39.36	1.00	47.69
10^{-3}	-22.63	0.40	14.30	8.73
5.10^{-3}	-26.37	-14.35	-18.04	- 6.02
10^{-2}	-25.92	-17.59	-17.59	- 9.26
0.05	-20.64	-18.31	-12.35	-10.02
0.10	-16.90	-15.98	- 8.73	- 7.81
0.15	-14.39	-13.97	- 6.42	- 6.00
0.20	-12.40	-12.19	- 4.71	- 4.50
0.25	-10.65	-10.54	- 3.30	- 3.19
0.30	- 8.98	- 8.93	- 2.02	- 1.97
0.35	- 7.31	- 7.29	- 0.80	- 0.78
0.40	- 5.58	- 5.57	0.45	0.46
0.45	- 3.76	- 3.75	1.77	1.78
0.50	- 1.81	- 1.81	3.19	3.19
0.55	0.29	0.29	4.75	4.75
0.60	2.55	2.55	6.47	6.47
0.65	5.02	5.02	8.40	8.40
0.70	7.73	7.73	10.58	10.58
0.75	10.74	10.74	13.07	13.07
0.80	14.15	14.15	15.98	15.98
0.85	18.18	18.18	19.52	19.52
0.90	23.27	23.27	24.15	24.15
0.95	30.90	30.90	31.33	31.33
0.99	46.00	46.00	46.08	46.08
0.999	65.61	65.61	65.62	65.62

TABLE II

X	Δf_2	Δf_3
10^{-5}	282.45	265.78
10^{-4}	231.78	215.12
10^{-3}	181.06	164.40
5.10^{-3}	145.45	128.78
10^{-2}	129.98	113.32
0.05	93.34	76.75
0.10	76.82	60.49
0.15	66.67	50.73
0.20	59.05	43.67
0.25	52.78	38.08
0.30	47.34	33.42
0.35	42.44	29.41
0.40	37.95	25.88
0.45	33.77	22.71
0.50	29.84	19.84
0.55	26.12	17.20
0.60	22.60	14.76
0.65	19.26	12.49
0.70	16.08	10.38
0.75	13.06	8.39
0.80	10.18	6.52
0.85	7.44	4.76
0.90	4.84	3.09
0.95	2.36	1.50
0.99	0.46	0.29
0.999	0.05	0.03

TABLE III

$1/X$	$\tilde{f}_2^+(1/X)$	$\tilde{f}_2^-(1/X)$	$\tilde{f}_3^+(1/X)$	$\tilde{f}_3^-(1/X)$
10^{-5}	308.46	381.95	300.12	373.61
10^{-4}	224.45	271.14	216.12	262.81
10^{-3}	158.43	181.46	178.70	173.13
5.10^{-3}	119.08	131.10	110.74	122.76
10^{-2}	104.06	112.39	95.73	104.06
0.05	72.70	75.03	64.40	66.73
0.10	59.92	60.84	51.76	52.68
0.15	52.28	52.70	44.31	44.73
0.20	46.65	46.86	38.39	38.60
0.25	42.13	42.24	34.78	34.89
0.30	38.36	38.41	31.40	31.45
0.35	35.13	35.15	28.61	28.63
0.40	32.37	32.38	26.33	26.34
0.45	30.01	30.02	24.48	24.49
0.50	28.03	28.03	23.03	23.03
0.55	26.41	26.41	21.95	21.95
0.60	25.15	25.15	21.23	21.23
0.65	24.28	24.28	20.89	20.89
0.70	23.81	23.81	20.96	20.96
0.75	23.80	23.80	21.46	21.46
0.80	24.33	24.33	22.50	22.50
0.85	25.62	25.62	24.28	24.28
0.90	28.11	28.11	27.24	27.24
0.95	33.26	33.26	32.83	32.83
0.99	46.46	46.46	46.37	46.37
0.999	65.66	65.66	65.65	65.65

A P P E N D I C E A

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EXEMPLE DU CALCUL DES VERTEX COUPES A DEUX BOUCLES

Dans cet appendice nous présentons en détail les différentes étapes des calculs des diagrammes à deux boucles. Nous nous intéressons au noyau d'évolution $\tilde{P}^{NS}(x,\alpha;\pm)$, non-singulet. Ce noyau est égal à la transformée de Mellin inverse des dimensions anormales des vertex coupés affectées d'un signe moins ($-\gamma^{NS}(\alpha;\pm)$).(voir chapitre II).

Nous effectuons les calculs dans le schéma de renormalisation dit "soustraction minimale" de 't Hooft et Veltman, c'est-à-dire nous utilisons la régularisation dimensionnelle en passant en dimension $n<4$ et nous soustrayons les parties de pôles $\frac{1}{n-4}$, $\frac{1}{(n-4)^2}$,... La quantité finie sera la quantité renormalisée. Dans ce schéma de renormalisation, les dimensions anormales à l'ordre α^2 sont les résidus des pôles simples multipliés par -2. Alors $\tilde{P}(x,\alpha)$ est égale à quatre fois le coefficient de $\frac{1}{\varepsilon}$ (où $\varepsilon = \frac{4-n}{2}$) du diagramme de probabilité (fig.1a,b).

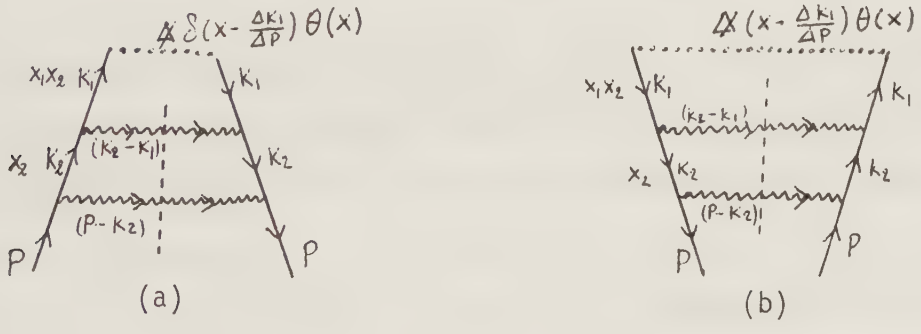

Fig. 1 : Diagrammes du second ordre contribuant aux noyaux d'évolution (a) $P_{qq}^{NS}(x,\alpha)$ et (b) $P_{qq}^{NS}(x,\alpha)$

Les règles de Feynman relatives aux vertex coupés sont données dans le chapitre VIII, page (204).

Nous effectuons les calculs en détail sur un exemple relativement simple pour en montrer les différentes étapes. Nous avons choisi le diagramme de fig.(1a) dans le cas des vertex coupés du genre espace et celui de fig.(1b) dans le cas du genre temps. Ces étapes sont les suivantes:

(i) Nous transformons la structure matricielle du numérateur en une somme de produits scalaires des impulsions k_1, k_2 et P, en le multipliant par $\frac{1}{4\Delta P}\not{P}$ et en prenant la trace. Nous travaillons dans la jauge de Feynman. On trouve:

$$-g^4 C_F^2 \cdot 4(1-\varepsilon_1)(1-\varepsilon_2)\Big[-k_1^2 P^2 x_2(1-x_1) + k_1^2 k_2^2 (1-x_2) + k_1^2 (P-k_2)^2 x_2 (1-x_1)$$
$$+ k_2^2 (P-k_1)^2 x_1 x_2 - k_2^2 (P-k_2)^2 x_1 x_2 - k_2^2 (k_1-k_2)^2 x_1 x_2 + k_2^4 x_1 x_2 - P^2 (k_1-k_2)^2 x_1 x_2$$
$$+ (P-k_2)^2 (k_1-k_2)^2 x_1 x_2 \Big] \cdot \delta(x - x_1 x_2) \cdot \theta(x) \tag{1}$$

où $$x_1 = \frac{\Delta k_1}{\Delta P} \quad , \quad x_2 = \frac{\Delta k_2}{\Delta P} \tag{2}$$

et $$2\varepsilon_1 = 4 - n_1 \quad , \quad 2\varepsilon_2 = 4 - n_2 \tag{3}$$

sont les différences de dimension $4 - n_i$ de chaque boucle i, relative à l'impulsion k_1 (n_1) et k_2 (n_2). Il est important de garder pour l'instant $\varepsilon_1 \neq \varepsilon_2$ comme nous le verrons par la suite. On a alors décomposé le numérateur en fonction des propagateurs k_1^2, k_2^2, $(k_1 - k_2)^2$, $(P-k_2)^2$ apparaissant au dénominateur, ce qui simplifie énormément les calculs. Dans le cas de l'exemple considéré, les termes du numérateur qui sont proportionnels aux lignes coupées sont nuls à cause des fonctions $\delta((P-k_2)^2)$ et $\delta((k_1-k_2)^2)$.

(ii) On effectue successivement les intégrations sur k_1 et k_2 en fixant x_1 et x_2 avec l'aide des identités suivantes:

(a) $$\int \frac{d^n k}{(2\pi)^n} \frac{\delta(x - \frac{\Delta k}{\Delta P})}{(-k^2 - i0)^A} \; abs \; \frac{1}{[-(P-k)^2 - i0]^B} \equiv \begin{cases} \to \{A, B\}_x \text{ genre-espace } k^2 < 0 \,, P^2 < 0 \\ \qquad\qquad 0 < x \leq 1 \\ \to \{\widetilde{A}, B\}_x \text{ genre-temps } k^2 > 0 \,, P^2 > 0 \\ \qquad\qquad 1 \leq x \leq \infty \end{cases}$$

avec

$$\{A,B\}_x = \frac{i\,\Gamma(A+B-\frac{\eta}{2})}{(4\pi)^{\eta/2}\,\Gamma(A)\Gamma(B)}\,(-P^2)^{\frac{\eta}{2}-A-B}\cdot x^{\frac{\eta}{2}-A-1}(1-x)^{\frac{\eta}{2}-B-1} \tag{5a}$$

$$\widetilde{\{A,B\}}_x = \frac{i e^{i\pi A}\,\Gamma(A+B-\frac{\eta}{2})}{(4\pi)^{\eta/2}\,\Gamma(A)\,\Gamma(B)}\,|P^2|^{\frac{\eta}{2}-A-B}\,x^{\frac{\eta}{2}-A-1}(x-1)^{\frac{\eta}{2}-B-1} \tag{5b}$$

(b)

$$\int \frac{d^\eta k}{(2\pi)^\eta}\,\frac{\delta(x-\frac{\Delta k}{\Delta P})\;k^\mu}{[-k^2-i0]^A}\;abs\;\frac{1}{[-(p-k)^2-i0]^B} \equiv \begin{cases} \to \{A,B\}^\mu_x \text{ genre-espace} & k^2<0\,,\ P^2\lesseqgtr 0 \\ & 0<x\leq 1 \\ \to \widetilde{\{A,B\}}^\mu \text{ genre-temps} & k^2>0\,,\ P^2>0 \\ & 1\leq x\leq\infty \end{cases} \tag{6}$$

avec

$$\overset{(\sim)}{\{A,B\}}^\mu_x = \overset{(\sim)}{\{A,B\}}_x \cdot x\left[p^\mu - \frac{P^2}{2\Delta P}\,\Delta^\mu\right] - \overset{(\sim)}{\{A-1,B\}}_x \frac{1}{x}\left(1-\frac{1-\frac{\eta}{2}}{A-1}(1-x)\right)\frac{\Delta^\mu}{2\Delta P} \tag{7}$$

Après les intégrations sur K_1 et K_2 on trouve le résultat ci-dessous:

$$C_F^2\,\frac{g^4}{(4\pi)^{4-\varepsilon_1-\varepsilon_2}}\cdot\frac{(-P^2)^{-\varepsilon_1-\varepsilon_2}}{\varepsilon_1(\varepsilon_1+\varepsilon_2)}\,4\,\Gamma(1+\varepsilon_1+\varepsilon_2)\Big[x_1^{-\varepsilon_1}(1-x_1)^{1-\varepsilon_1}x_2^{-\varepsilon_1-\varepsilon_2}(1-x_2)^{1-\varepsilon_2} - \varepsilon_1$$

$$-(\varepsilon_1+\varepsilon_2)(1-x_1)+\varepsilon_1(1-x_1)x_2\Big]\,\delta(x-x_1x_2)\cdot\theta(x)\,\theta(1-x). \tag{8}$$

Dans la dernière expression nous avons negligé les termes qui sont finis dans la limite $\varepsilon_1=\varepsilon_2\to 0$.

(iii) On trouve facilement la contribution du diagramme réduit qui contient le contreterme à une boucle (figs. 2a,2b)

Fig. 2 : Les diagrammes réduits à partir de la figure 1 et contenant les contretermes à une boucle. La contribution des contretermes est la partie de pôle des sous diagrammes hachurée.

à partir de l'expression (8). En effet le pôle $1/\varepsilon_1$ indique qu'il existe une sous-divergence sur la variable K_1 . Alors la contribution du diagramme réduit fig. 2a. sera égale à

$$-\frac{C_F^2 g^4}{(4\pi)^{4-\varepsilon_2}} \frac{(-p^2)^{-\varepsilon_2}}{\varepsilon_1 \cdot \varepsilon_2} 4\Gamma(1+\varepsilon_2)\left[(1-x_1)x_2^{-\varepsilon_2}(1-x_2)^{1-\varepsilon_2} - \varepsilon_2(1-x_1)\right]\delta(x-x_1x_2)\theta(x)\theta(1-x) \qquad (9)$$

c'est-à-dire le résidu du pôle $\frac{-1}{\varepsilon_1}$ de l'expression (8).

L'utilité de garder $\varepsilon_1 \neq \varepsilon_2$ est manifeste: cela sert à isoler les sous-divergences et à trouver sans avoir à répéter les calculs des contributions des divers contretermes.

(iv) On identifie ε_1 et ε_2 avec ε et on effectue la somme des contributions particulières (8) et (9). On trouve comme résultat l'expression suivante:

$$C_F^2 \frac{g^4}{(4\pi)^4} \frac{2}{\varepsilon^2}\left[-(1-x_1)(1-x_2) - \varepsilon(1-x_1)(1-x_2)\left[\log x_1 - \log(1-x_1) + \log(1-x_2) - 1\right]\right]\delta(x-x_1x_2)\theta(x)\theta(1-x) \qquad (10)$$

(v) Les intégrations sur x_1 et x_2 de 0 à 1 représentent un produit de convolution à cause de la présence de $\delta(x-x_1x_2)$. En effet:

$$\int_0^1 dx_1 \int_0^1 dx_2\ \delta(x-x_1x_2)\varphi(x_1)f(x_2) \equiv \varphi(x) \otimes f(x) \qquad (11)$$

L'expression (10) devient:

$$C_F^2 \frac{g^4}{(4\pi)^4} \frac{2}{\varepsilon^2}\left[-(1-x)\otimes(1-x) - \varepsilon(1-x)\otimes(1-x)(\log x - 1)\right]\theta(x)\theta(1-x) \qquad (12)$$

Les termes $-\log(1-x_2)$ et $\log(1-x_1)$ se simplifient car :

$$(1-x)\otimes(1-x)\log(1-x) = (1-x)\log(1-x)\otimes(1-x) \qquad (13)$$

(vi) La contribution à $P(x)$ est égale à quatre fois le coefficient de $1/\varepsilon$. En effectuant les convolutions qui apparaissent dans ce coefficient à l'aide des relations données dans l'Appendice B, on trouve:

$$P(x) = 8C_F^2\frac{g^4}{(4\pi)^4}\left(\frac{1}{2}(1+x)\log^2 x + 2\log x + 3(1-x)\right) \qquad (14)$$

En effectuant les mêmes étapes pour $\tilde{P}(x)$ on trouve le résultat de la continuation analytique

$$P(x) = -\tilde{P}(x)\ ;\ x<1 \qquad (15)$$

Dans le cas général, certaines coupures d'un diagramme ont des divergences infrarouges. On a trouvé que les divergences infrarouges disparaissent quand on ajoute toutes les coupures relatives à chaque diagramme. En régularisant les divergences infrarouges dimensionnellement, il n'est pas possible de les confondre avec les divergences ultraviolettes car les divergences infrarouges apparaissent toujours après l'intégration sur K_i^+ ; par contre les divergences ultraviolettes sont le résultat de l'intégration sur K_i^+ à K_i^+ fixé. D'une façon générale les divergences infrarouges s'expriment toujours comme:

$$\frac{1}{(1-x_i)^{1-\varepsilon}} = \frac{1}{\varepsilon}\delta(1-x_i) + \frac{1}{(1-x_i)_+} + \varepsilon\left(\frac{\log(1-x_i)}{1-x_i}\right)_+ + \cdots \qquad (16)$$

La quantité $(1-x_i)$ représente la fraction d'impulsion "+" d'un gluon dans l'état final. Si $1-x_i = 0$ on a la production d'un gluon "mou". Le pôle $1/\varepsilon$ de l'expression (16) s'annule quand on ajoute la contribution d'une boucle fermée contenant un gluon "mou" (voir l'exemple qui est présenté dans le chapitre VIII, page (181)).

Pour compléter cet appendice nous donnons la formule générique pour une boucle fermée qui ne contient pas de lignes coupées.

$$\int \frac{d^n K}{(2\pi)^n} \frac{\delta(x - \frac{\Delta K}{\Delta P})}{(-K^2 - i0)^A (-(P-K)^2 - i0)^B} \equiv (A,B)_x \quad , \quad P^2 < 0 \quad , \quad \frac{\Delta K}{\Delta P} > 0 \tag{17a}$$

$$\int \frac{d^n K}{(2\pi)^n} \frac{\delta(x - \frac{\Delta K}{\Delta P}) \quad K^\mu}{(-K^2 - i0)^A (-(P-K)^2 - i0)^B} \equiv (A,B)_x^\mu \quad , \quad P^2 < 0, \quad \frac{\Delta K}{\Delta P} > 0 \tag{17b}$$

On trouve que

$$(A,B)_x = \{A,B\}_x \qquad \text{et} \qquad (A,B)_x^\mu = \{A,B\}_x^\mu \tag{18}$$

ce qui signifie que $(A,B)_x$ est identique à ce qu'on obtient en ne gardant que la partie absorptive du propagateur $-(P-K)^2 - i0$, dans le cas $K^2 < 0$, $P^2 < 0$.

APPENDICE' B

Nous présentons dans cet appendice les transformées de Mellin (inverses) et les produits de convolution pour certaines fonctions clés qui sont très utiles dans les calculs et l'analyse des ordres supérieurs. Nous présentons également le prolongement analytique de ces fonctions clés dans l'espace des x ainsi que dans l'espace des moments.

1.- Transformation de Mellin

$$\varphi(x) \longleftrightarrow \langle\varphi\rangle_\sigma = \int_0^1 dx\, x^{\sigma-1}\varphi(x)$$

$$\varphi(x)\,x^a \longleftrightarrow \langle\varphi\rangle_{\sigma+a}$$

$$\varphi(x)\log^k\frac{1}{x} \longleftrightarrow \left(\frac{d}{d\sigma}\right)^k\langle\varphi\rangle_\sigma$$

$$\delta(1-x) \longleftrightarrow 1$$

$$x^a \longleftrightarrow \frac{1}{\sigma+a}$$

$$x^a\log^\beta\frac{1}{x} \longleftrightarrow \Gamma(\beta+1)/(\sigma+a)^{\beta+1}$$

$$\log(1-x) \longleftrightarrow \frac{-1}{\sigma}S_1(\sigma) \qquad \left\| S_m(\sigma)=\sum_{k=1}^{\sigma}\frac{1}{k^m}\ ;\ \sigma \text{ entier}\right.$$

$$\log^2(1-x) \longleftrightarrow \frac{1}{\sigma}\left(S_1^2(\sigma)+S_2(\sigma)\right)$$

$$\log^3(1-x) \longleftrightarrow \frac{-1}{\sigma}\left(S_1^3(\sigma)+3S_1(\sigma)S_2(\sigma)+2S_3(\sigma)\right)$$

$$\log(1+x) \longleftrightarrow \frac{-(-)^\sigma}{\sigma}\left(\mathcal{S}_1(\sigma)+(1-(-)^\sigma)\log 2\right) \qquad \left\| \mathcal{S}_m(\sigma)=\sum_{k=1}^{\sigma}\frac{(-)^m}{k^m}\ ;\sigma \text{ entier}\right.$$

$$\log(1-x)\log x+\frac{\pi^2}{6} \longleftrightarrow \frac{1}{\sigma^2}S_1(\sigma)+\frac{1}{\sigma}S_2(\sigma)$$

$$\log(1+x)\log x \quad \longleftrightarrow \quad \frac{(-)^\sigma}{\sigma}\left(S_2(\sigma)+\frac{\pi^2}{12}\right)+\frac{(-)^\sigma}{\sigma^2}\left(S_1(\sigma)+(1-(-)^\sigma)\log 2\right)$$

$$Li_2(x)-\frac{\pi^2}{6} \quad \longleftrightarrow \quad \frac{-1}{\sigma^2}S_1(\sigma) \qquad \left\| \; Li_2(x) = -\int_0^x \frac{d\xi}{\xi}\log(1-\xi)\right.$$

$$Li_2(1-x)-\frac{\pi^2}{6} \quad \longleftrightarrow \quad \frac{-1}{\sigma}S_2(\sigma)$$

$$Li_2(-x)+\frac{\pi^2}{12} \quad \longleftrightarrow \quad \frac{-(-)^\sigma}{\sigma^2}\left(S_1(\sigma)+(1-(-)^\sigma)\log 2\right)$$

$$\left(\frac{x}{1-x}\right)_+ \quad \longleftrightarrow \quad 1-S_1(\sigma)$$

$$\left(\frac{x\log^{K-1}\frac{1}{x}}{1-x}\right)_+ \quad \longleftrightarrow \quad K!\left(1-S_K(\sigma)\right)$$

$$\left(\frac{x\log(1-x)}{1-x}\right)_+ \quad \longleftrightarrow \quad \frac{1}{2}\left(S_1^2(\sigma)+2S_2(\sigma)\right)-1$$

$$\left(\frac{x\log^2(1-x)}{1-x}\right)_+ \quad \longleftrightarrow \quad \frac{1}{3}\left(S_1^3(\sigma)+3S_1(\sigma)S_2(\sigma)+2S_3(\sigma)\right)$$

$$\left(\frac{x\left(Li_2(x)-Li_2(1)\right)}{1-x}\right)_+ \quad \longleftrightarrow \quad \hat{S}(\sigma)-1 \qquad \left\| \; \hat{S}(\sigma)=\sum_{m=1}^{\sigma}\frac{1}{m}S_2(m)\right.$$

$$\left(\frac{x\left(Li_2(1-x)-Li_2(1)\right)}{1-x}\right)_+ \quad \longleftrightarrow \quad S_1(\sigma)S_2(\sigma)+S_3(\sigma)-\hat{S}(\sigma)-1$$

$$\left(\frac{x\left(\log x\log(1-x)+\frac{\pi^2}{6}\right)}{1-x}\right)_+ \quad \longleftrightarrow \quad -S_1(\sigma)S_2(\sigma)-S_3(\sigma)+2$$

$$\frac{x^\alpha-x^\beta}{\log x} \quad \longleftrightarrow \quad \log\frac{\sigma+\alpha}{\sigma+\beta}$$

$$\left(\frac{x\left(\frac{1}{2}\log^2 x-2\log x\log(1+x)-2\left(Li_2(-x)+\pi^2/12\right)\right)}{1+x}\right) \quad \longleftrightarrow \quad (-)^\sigma\left(\frac{1}{2}Li_3(1)-\frac{\pi^2}{6}S_1(\sigma)-S_3(\sigma)-2S_1(\sigma)S_2(\sigma)+2\tilde{S}(\sigma)\right)$$

$$\left\| \; \tilde{S}(\sigma)=\sum_{m=1}^{\sigma}\frac{(-)^m}{m^2}S_1(m)\right.$$

Dans le cas d'annihilation où $\tilde{\varphi}(x)$ est défini pour $X \geq 1$, on utilise les mêmes formules que celles présentées ci-dessus avec la substitution $f(z) = \tilde{\varphi}(\frac{1}{z})$, $0 \leq z \leq 1$. En effet :

$$\langle \tilde{\varphi} \rangle_\sigma \equiv \int_1^\infty dx\, x^{-\sigma-1} \tilde{\varphi}(x) = \int_0^1 dz\, z^{\sigma-1} \tilde{\varphi}(\tfrac{1}{z}) = \int_0^1 dz\, z^{\sigma-1} f(z) \equiv \langle f \rangle_\sigma$$

2. - Produit de convolution entre deux fonctions :

Par définition, le produit de convolution de φ et de f est :

$$\varphi(x) \otimes f(x) = \pm \int_x^1 \frac{dy}{y} \varphi(\tfrac{x}{y}) f(y) = \pm \int_x^1 \frac{dy}{y} \varphi(y) f(\tfrac{x}{y})$$

avec le signe + dans le cas de diffusion ($0 \leq X \leq 1$) et avec le signe - dans le cas d'annihilation ($1 \leq X < a$)

$$\varphi(x) \otimes f(x) \longrightarrow \int_x^1 \frac{dy}{y} \varphi(\tfrac{x}{y}) f(y)$$

$$x^a . [\varphi(x) \otimes f(x)] \longrightarrow (x^a \varphi(x)) \otimes (x^a f(x))$$

$$\log x . [\varphi(x) \otimes f(x)] \longrightarrow (\log x\, \varphi(x)) \otimes f(x) + \varphi(x) \otimes (\log x\, f(x))$$

$$\varphi(x) \otimes \delta(1-x) \longrightarrow \varphi(x)$$

$$\varphi(x) \otimes \delta'(1-x) \longrightarrow -x \frac{d}{dx} \varphi(x)$$

$$x^\alpha \otimes x^\beta \longrightarrow \frac{1}{\beta-\alpha} (x^\alpha - x^\beta) \quad ; \quad \alpha \neq \beta$$

$$x^\alpha \otimes x^\alpha \longrightarrow -x^\alpha \log x$$

$$x^\alpha \otimes \log x \longrightarrow \frac{1}{\alpha^2} (1 - x^\alpha) + \frac{1}{\alpha} \log x$$

$$1 \otimes \log(1-x) \longrightarrow Li_2(x) - \frac{\pi^2}{6}$$

$$x \otimes \log(1-x) \longrightarrow x\log x + (1-x)\log(1-x)$$

$$1 \otimes x\log(1-x) \longrightarrow x - 1 + (1-x)\log(1-x)$$

$$x^2 \otimes \log(1-x) \longrightarrow \frac{1}{2}(1-x^2)\log(1-x) + \frac{1}{2}x^2\log x - \frac{1}{2}x(1-x) \,.$$

$$1 \otimes x^2\log(1-x) \longrightarrow \frac{1}{2}(1-x^2)\log(1-x) - \frac{1}{4}(1-x)(3+x)$$

$$1 \otimes \frac{1}{(1-x)_+} \longrightarrow \log(1-x) - \log x$$

$$x \otimes \frac{1}{(1-x)_+} \longrightarrow x\log(1-x) - x\log x + (1-x)$$

$$1 \otimes \left(\frac{\log x}{1-x}\right)_+ \longrightarrow \log x \log(1-x) + Li_2(x) - \frac{1}{2}\log^2 x$$

$$\log x \otimes \left(\frac{1}{1-x}\right)_+ \longrightarrow -Li_2(x) + \frac{\pi^2}{6} - \frac{1}{2}\log^2 x$$

$$x \otimes \left(\frac{\log x}{1-x}\right)_+ \longrightarrow x\log x\log(1-x) + xLi_2(x) - \frac{1}{2}x\log^2 x + \log x + (1-x)$$

$$x\log x \otimes \left(\frac{1}{1-x}\right)_+ \longrightarrow -xLi_2(x) + x\frac{\pi^2}{6} - \frac{1}{2}x\log^2 x - (1-x) - x\log x$$

$$1 \otimes \left(\frac{\log(1-x)}{1-x}\right)_+ \longrightarrow \frac{1}{2}\log^2(1-x) + Li_2(x) - \frac{\pi^2}{6}$$

$$\log(1-x) \otimes \left(\frac{1}{1-x}\right)_+ \longrightarrow \log^2(1-x) - \log x\log(1-x) - \frac{\pi^2}{6}$$

$$x \otimes \left(\frac{\log(1-x)}{1-x}\right)_+ \longrightarrow \frac{1}{2}x\log^2(1-x) + xLi_2(x) - x\frac{\pi^2}{6} + (1-x)\log(1-x) + x\log x$$

$$x\log(1-x) \otimes \left(\frac{1}{1-x}\right)_+ \longrightarrow x\log^2(1-x) - x\log x\log(1-x) - x\frac{\pi^2}{6} + (1-x)\log(1-x) - (1-x)$$

3. - Prolongement analytique dans l'espace des X

$$\varphi(x) = -\,\text{Partie Réelle}\left[\tilde{\varphi}(x+i\omega)\,;\ x<1\right] \tag{3.1}$$

Dans le cas où $\tilde{\varphi}(x)$ est une fonction analytique, le résultat du membre droit de l'équation ci-dessus, sera la partie réelle du prolongement analytique dans la région où $X<1$.

Dans le cas où $\tilde{\varphi}(x)$ est une distribution, le prolongement analytique sera défini à partir de la représentation régularisée de cette distribution :

$$\frac{1}{(x-1+i\omega)^{1-\varepsilon}} = \frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(x-1)_+} + \varepsilon\left(\frac{\log(x-1)}{x-1}\right)_+ + \frac{1}{2}\varepsilon^2\left(\frac{\log^2(x-1)}{x-1}\right)_+ + \cdots \tag{3.2a}$$

$$-\text{Re}\ \Big|\ x<1$$

$$\text{Re}\ \frac{e^{i\pi\varepsilon}}{(1-x)^{1-\varepsilon}} = \frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left[\left(\frac{\log(1-x)}{1-x}\right)_+ - \frac{\pi^2}{2}\delta(1-x)\right] + \frac{1}{2}\varepsilon\left(\frac{\log^2(1-x)-\pi^2}{1-x}\right)_+ + \cdots$$

Après l'identification des différentes puissances en ε on trouve :

$$\begin{aligned}
\tilde{\varphi}(x) &\longleftrightarrow \varphi(x)\\
\delta(1-x) &\longleftrightarrow \delta(1-x)\\
\frac{1}{(x-1)_+} &\longleftrightarrow \frac{1}{(1-x)_+}\\
\left(\frac{\log(x-1)}{x-1}\right)_+ + \frac{\pi^2}{2}\delta(1-x) &\longleftrightarrow \left(\frac{\log(1-x)}{1-x}\right)_+\\
\left(\frac{\log^2(x-1)+\pi^2}{x-1}\right)_+ &\longleftrightarrow \left(\frac{\log^2(1-x)}{1-x}\right)_+
\end{aligned} \tag{3.3}$$

4. - Prolongement analytique dans l'espace des moments

$$\langle\varphi\rangle_\sigma = \text{Partie Réelle}\left[\langle\tilde{\varphi}\rangle_{-\sigma}\right] \tag{4.1}$$

Pour bien définir l'opération $-\sigma$ il faut respecter le prolongement analytique que nous avons trouvé dans l'espace des X.

La généralisation correcte s'obtient à partir de la régularisation suivante valable pour la fonction $\psi(x) = \frac{d}{dx} \log \Gamma(x)$

$$\psi(-\sigma+1) \xrightarrow{\text{Regular.}} \psi(-\sigma+1) - \pi \operatorname{ctg} \pi\sigma + i\pi = \psi(\sigma) + i\pi \tag{4.2}$$

Cette régularisation est en accord avec le prolongement analytique des distributions obtenu dans l'espace des X. La régularisation des dérivées d'ordres plus élevés de ψ est la suivante

$$\psi^{(k)}_{reg.}(-\sigma+1) = (-)^k \psi^{(k)}(\sigma) + i\pi \delta_{k,0} \tag{4.3}$$

En utilisant cette régularisation nous pouvons définir le prolongement analytique $\sigma \to -\sigma$ pour les expressions suivantes :

Etant données

$$S_{\ell+1}(\sigma) = \frac{(-)^\ell}{\ell!}\left[\psi^{(\ell)}(\sigma+1) - \psi^{(\ell)}(1)\right] \xrightarrow[\text{entier}]{\sigma} \sum_{m=1}^{\sigma} \frac{1}{m^{\ell+1}} \tag{4.4a}$$

$$\mathcal{S}_{\ell+1}(\sigma) = \frac{(-)^\ell}{\ell!\, 2^{\ell+1}}\left[\psi^{(\ell)}\left(\tfrac{1}{2}\right) - \psi^{(\ell)}(1) + (-)^\sigma\left(\psi^{(\ell)}\left(\tfrac{\sigma}{2}+1\right) - \psi^{(\ell)}\left(\tfrac{\sigma}{2}+\tfrac{1}{2}\right)\right)\right]$$

$$\xrightarrow[\text{entier}]{\sigma} \sum_{m=1}^{\sigma} \frac{(-)^m}{m^{\ell+1}} \tag{4.4b}$$

D'après la régularisation (4.3) $S_\ell(-\sigma)$ et $\mathcal{S}_\ell(-\sigma)$ sont :

$$S_\ell(-\sigma) \xrightarrow{\text{Regul.}} (-)^{\ell-1} S_\ell(\sigma-1) + ((-)^\ell + 1) \sum_{m=1}^{\infty} \frac{1}{m^\ell} + i\pi\delta_{\ell,1} \tag{4.5a}$$

$$\mathcal{S}_\ell(-\sigma) \xrightarrow{\text{Regul.}} (-)^\ell \mathcal{S}_\ell(\sigma-1) + ((-)^{\ell+1} + 1) \sum_{m=1}^{\infty} \frac{(-)^m}{m^\ell} \tag{4.5b}$$

La partie imaginaire qui apparaît dans le prolongement de $S_1(\sigma)$ est facile à comprendre. En effet, pour $Re\,\sigma \to \infty$, $S_1(\sigma)$ se comporte comme $\log\sigma$. En effectuant l'opération $\sigma \to -\sigma$, $\log\sigma \to \log\sigma + i\pi$.

Nous donnons ci-dessous le prolongement analytique (4.1) pour certaines fonctions de σ qui apparaissent dans les expressions des dimensions anormales et dans les fonctions coefficients des vertex coupés.

"genre espace"		"genre temps"
$S_1(\sigma)$	$\longleftrightarrow$	$S_1(\sigma-1)$
$S_2(\sigma)$	$\longleftrightarrow$	$-S_2(\sigma-1)+\frac{\pi^2}{3}$
$S_3(\sigma)$	$\longleftrightarrow$	$S_3(\sigma-1)$
$S_1'(\frac{\sigma}{2})-S_1(\sigma)\equiv \mathcal{S}_1(\sigma)$	$\longleftrightarrow$	$-\mathcal{S}_1(\sigma-1)-2\log 2$
$\frac{1}{2}S_2'(\frac{\sigma}{2})-S_2(\sigma)\equiv \mathcal{S}_2(\sigma)$	$\longleftrightarrow$	$\mathcal{S}_2(\sigma-1)$
$\frac{1}{4}S_3'(\frac{\sigma}{2})-S_3(\sigma)\equiv \mathcal{S}_3(\sigma)$	$\longleftrightarrow$	$-\mathcal{S}_3(\sigma-1)-\frac{3}{2}Li_3(1)$
$S_1^2(\sigma)$	$\longleftrightarrow$	$S_1^2(\sigma-1)+\pi^2$
$\tilde{S}(\sigma)\equiv\sum_{m=1}^{\infty}\frac{(-)^m}{m^2}S_1(m)$	$\longleftrightarrow$	$\tilde{S}(\sigma-1)+\mathcal{S}_3(\sigma-1)+\frac{3}{2}Li_3(1)$